Controllability results of linear difference equations

Difference equations are well known to be important tools in the analysis of hyperbolic systems of PDEs, since they provide a handy representation for some simplified dynamics. This talk considers the controllability of the difference equation

\[ x(t) = \sum_{j=1}^{N} A_j x(t - L_j) + Bu(t) \]

where \( x(t) \in \mathbb{R}^d, u(t) \in \mathbb{R}^m \) is the control, \( A_1, \ldots, A_N \) and \( B \) are matrices of appropriate dimensions, and \( L_1, \ldots, L_N \) are positive delays. We present necessary and sufficient conditions on relative controllability, which consists on controlling the final state \( x(T) \), in terms of some matrix coefficients computed from \( A_1, \ldots, A_N \) generalizing the usual Kalman controllability criterion. We also have results on exact and approximate \( L_2 \) controllability, which are complete in the case \( N = d = 2, m = 1 \). Our approach relies in an explicit formula for solutions, which has the advantage of providing criteria for all positive delays \( L_1, \ldots, L_N \), i.e., with no assumption of commensurability.