Welcome to the

## Workshop Stability and Control of Infinite-Dimensional Systems

**October 12<sup>th</sup> – 14<sup>th</sup>, 2016 in Passau, Germany** 

<u>Organizers:</u> Sergey Dashkovskiy Birgit Jacob Andrii Mironchenko Fabian Wirth

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Mohamadreza Ahmadi (University of Oxford)

## Title:

PDE analysis and polynomial optimization

## Abstract:

The analysis of PDE systems in many cases leads to a set of integral inequalities. We present a method to reformulate the problem of checking the non-positivity of these integral inequalities into checking the non-positivity of matrix inequalities over some domain. In the case of polynomial data, we solve the associated matrix inequalities by polynomial optimization using the sum-of-squares (SOS) programming techniques. This allows us to cast a class of PDE analysis problems into SOS programs that can be solved by available computational tools. We then show how this methodology can be used to address problems in stability analysis, input-output analysis, fluid flow analysis and safety verification.

## Author:

Vincent Andrieu (University of Lyon)

## Title:

Regulation by integral controller for quasi-linear hyperbolic PDE

#### Abstract:

This talk deals with the control and regulation by integral controllers for the nonlinear systems governed by scalar quasi-linear hyperbolic partial differential equations. Both the control input and the measured output are located on the boundary. The closed loop stabilization of the linearized model with the designed integral controller is proved first by using the method of spectral analysis and then by the Lyapunov direct method. Based on the elaborated Lyapunov function we prove local exponential stability of the nonlinear closed-loop system with the same controller. The output regulation to the set-point with zero static error by the integral controller is shown upon the nonlinear system.

Yacine Chitour (LSS-Supelec, Gif-sur-Yvette, France)

#### Title:

Controllability results of linear difference equations

#### Abstract:

Difference equations are well known to be important tools in the analysis of hyperbolic systems of PDEs, since they provide a handy representation for some simplified dynamics. This talk considers the controllability of the difference equation  $x(t) = \sum_{j=1}^{N} A_j x(t - L_j) + Bu(t)$  where  $x(t) \in \mathbb{R}^d$ ,  $u(t) \in \mathbb{R}^m$  is the control,  $A_1, ..., A_N$  and B are matrices of appropriate dimensions, and  $L_1, ..., L_N$  are positive delays. We present necessary and sufficient conditions on *relative* controllability, which consists on controlling the final state x(T), in terms of some matrix coefficients computed from  $A_1, ..., A_N$  generalizing the usual Kalman controllability criterion. We also have results on exact and approximate  $L_2$  controllability, which are complete in the case N = d = 2, m = 1. Our approach relies in an explicit formula for solutions, which has the advantage of providing criteria for all positive delays  $L_1, ..., L_N$ , i.e., with no assumption of commensurability.

#### Author:

Sergey Dashkovskiy (University of Wuerzburg)

#### Title:

Stability of systems with dynamics depending on the maximum of solution over a prehistory

#### Abstract:

Systems with dynamics that depends on the maximum of the solution over some prehistory time interval are a special subclass of nonlinear infinite dimensional systems. Certain stability theory for them exists in the literature. So far such systems were considered without inputs. Our aim is to investigate their properties in case of existing input signals. We will consider some known results for such systems and discuss about several open problems related to stability, robustness and interconnections.

Lars Grüne (University of Bayreuth, Germany)

#### Title:

Stability results for model predictive control schemes for infinite-dimensional systems

#### Abstract:

The first part of the talk presents results for the stability analysis of stabilizing model predictive control (MPC) schemes for infinite dimensional systems. The application of the results will be illustrated for several types of PDE governed control systems, including reaction-diffusion equations and the Fokker-Planck PDE. The second part will sketch recent attempts to extend these results to economic MPC schemes.

The presented results are based on joint work with Nils Altmüller, Arthur Fleig and Marleen Stieler.

## Author:

Birgit Jacob (University of Wuppertal)

#### Title:

On input-to-state-stability and integral input-to-state-stability for parabolic boundary control systems

#### Abstract:

It is well known, that the notions *input-to-state stability* and *integral input-to-state stability* are equivalent for finite-dimensional linear systems and even for in infinite dimensional systems of the form

$$\frac{d}{dt}x = Ax(t) + Bu(t); \quad x(0) = x_0;$$
(1)

where *A* generates a  $C_0$ -semigroup on a Banach space *X* and *B* is a linear bounded operator from another Banach space *U* to *X*. In this talk we study the relation of these two notions for linear infinite-dimensional systems of the form (1) with a (possibly) unbounded control operator. We show that integral input-to-state stability implies input-to-state stability. For parabolic diagonal systems, the notions are even equivalent. In particular, a simple criterion for input-to-state stability is derived.

This is joint work with R. Nabiullin (Wuppertal), J.R. Partington (Leeds) and F. Schwenninger (Hamburg).

Oleksiy Kapustyan (Taras Shevchenko National University of Kyiv, Ukraine)

#### Title:

Global attractors of infinite-dimensional dynamical systems and their stability under multi-valued perturbations

#### Abstract:

Global attractors - compact invariant stable uniformly attracting sets - are one of the most important objects for describing the long-time behavior of infinite-dimensional dynamical systems. But in many dissipative evolutionary systems we face the problem of a lack of uniqueness of solutions. In the report we discuss some examples including 3D Navier-Stokes systems, evolutionary inclusions and multidimensional systems. One of the possible ways to deal with such problems is to pass to multi-valued semigroups. We propose a generalization of the classical theory of global attractors to the case of multi-valued infinite-dimensional dynamical systems. As an application we consider the problem of stability of global attractors under multivalued perturbation.

## Author:

lasson Karafyllis (National Technical University of Athens, Greece)

#### Title:

ISS in different norms for 1-D parabolic PDEs with boundary disturbances

#### Abstract:

For 1-D parabolic PDEs with disturbances at both boundaries and distributed disturbances we provide ISS estimates in various norms. Due to the lack of an ISS Lyapunov functional for boundary disturbances, the proof methodology uses (i) an eigenfunction expansion of the solution, and (ii) a finite-difference scheme. The ISS estimate for the sup-norm leads to a refinement of the well-known maximum principle for the heat equation. The obtained results are applied to quasi-static thermoelasticity models that involve nonlocal boundary conditions. Small-gain conditions that guarantee the global exponential stability of the zero solution for such models are derived. Finally, the obtained results are applied to parabolic PDEs under observerbased boundary feedback control.

Miroslav Krstic (University of California, San Diego, CA, USA)

#### Title:

Control of PDE-ODE cascades

#### Abstract:

In this talk I will review control designs for cascades of transport, wave, and heat equations into ODE systems. The transport-ODE cascade represents the classical problem of input delay, whereas the other PDE-ODE cascades arise in systems such as oil drilling and melting/crystallization.

## Author:

Hartmut Logemann (University of Bath, UK)

#### Title:

Stability of infinite-dimensional Lur'e systems and integral control

#### Abstract:

Lur'e systems are feedback interconnections of linear dynamical systems and static nonlinearities. In the work to be presented, we will consider forced Lur'e systems, the underlying linear system of which is assumed to be a well-posed infinite-dimensional system. The input and output spaces of the well-posed system are product spaces and the nonlinear feedback interconnects one "component "of the output to one "component" of the input. We will present and discuss stability results of ISS type which are reminiscent of certain classical finite-dimensional absolute stability theorems. The stability results will be applied in the context of an integral control problem, namely that of set-point tracking for stable well-posed linear infinite-dimensional systems in the presence of input nonlinearities (including saturation).

The material presented in the talk is based on joint work with Chris Guiver (Bath) and Mark Opmeer (Bath).

Frédéric Mazenc (Inria Saclay, France)

## Title:

New trajectory based approach for systems with delay: application to the reduction model technique

## Abstract:

We propose a new technique for stability analysis for nonlinear dynamical systems with delays and possible discontinuities. In contrast with Lyapunov based approaches, our trajectory based approach involves verifying certain inequalities along solutions of auxiliary systems. It applies to a wide range of systems, notably time-varying systems with time-varying delay, ODE coupled with difference equations, and networked control systems with delay. It relies on the input-to-state stability notion, and yields input-to-state stability with respect to uncertainty. As an application of this technique, we solve a stabilization problem for linear continuous time time-varying systems with bounded time-varying delays. We provide a novel reduction model approach that ensures global exponential stabilization of linear systems with a time-varying pointwise delay in the input, which allows the delay to be discontinuous and uncertain. Finally, we provide an alternative to the reduction model method, based on a different dynamic extension. Our main results use upper bounds on an integral average involving the delay.

## Author:

Thomas Meurer (University of Kiel, Germany)

## Title:

Tracking control for PDEs in single and higher dimensions

## Abstract:

Partial differential equations (PDEs) arise in a broad variety for the mathematical description of dynamic systems. Examples include smart structures, fixed-bed reactors, fluid flow, and even interconnected multi-agent systems. Their operation relies on the integration of suitable control strategies, e.g., to suppress vibrations, to avoid and detect hot spots or to stabilize formations. Complementing the feedback stabilization in the last years systematic motion planning and tracking control concepts to impose a desired transient behavior have gained increasing interest both in control theory and applications.

This contribution addresses recent developments for the tracking control of PDE systems in single and higher dimensional spatial domains. This will include flatness– based methods for motion planning and feedforward control as well as backstepping–based feedback control and observer design. It will be shown that their combination leads to stabilizing tracking controllers, even for certain classes of nonlinear PDEs, to achieve a prescribed desired spatial–temporal evolution of the state variables. The applicability and control performance of the developed design techniques will be illustrated in numerical simulations for diffusion–convection– reaction systems and, by exploiting a continuum approach, for coupled multi–agent systems.

Andrii Mironchenko (University of Passau)

## Title:

Input-to-state stability of distributed parameter systems: characterizations and counterexamples

#### Abstract:

In this talk we consider input-to-state stability (ISS) of nonlinear evolution equations in Banach spaces. For this class of systems we characterize local and global ISS properties in terms of Lyapunov functions and other stability properties. At the same time we show by means of counterexamples that some important criteria for ISS known for ordinary differential equations are no more valid in infinite dimensions.

#### Author:

Svyatoslav Pavlichkov (National University of Singapore)

#### Title:

Decentralized stabilization of interconnected finite-dimensional and infinite-dimensional systems

#### Abstract:

We discuss the applications of small-gain theorems to design of decentralized controllers for interconnected multi-agent systems. We focus on possible extensions of the existing approaches to the case of networks with infinite set of interconnected agents. Our goal is to provide a suitable gain assignment and to define a suitable normed space as the state space of the entire network in order to ensure the applicability of the classical scheme to the infinite-dimensional case.

Pierdomenico Pepe (University of L'Aquila, Italy)

#### Title:

Practical stabilization of nonlinear retarded systems: continuous-time and sampled data controllers

#### Abstract:

In this talk Sontag's universal practical stabilizer is presented for nonlinear systems described by Retarded Functional Differential Equations. Sufficient conditions for practical stabilization and input-to-state practical stabilization are shown, in terms of invariantly differentiable control Lyapunov-Krasovskii functionals. Results concerning sampled-data controllers are also presented. It is shown that suitable steepest descent state feedbacks (continuous or not) induced by control Lyapunov-Krasovskii functionals, yield practical stabilization with arbitrary small final target ball of the origin, when applied to the system by sampling and holding with sufficiently fast sampling. It is shown that the first order spline approximation method can be successfully used in order to implement (infinite dimensional) state feedbacks by a finite number of (finite dimensional) samples of the system variable. Examples of application are shown.

#### Author:

Christophe Prieur (Gipsa-lab, Grenoble, France)

## Title:

Using saturated controls for the control of PDEs

#### Abstract:

The general problem under consideration in this talk will be the use of saturated controller for PDEs. Two kinds of equations will be considered in this talk: the linear wave equation (with either boundary controller or in-domain control), and the nonlinear Korteweg-de Vries equation (with internal control). For both equations, saturating control laws will be designed. By taking into consideration the presence of amplitude-limited controls, we will apply nonlinear semigroup theory and Lyapunov techniques, among other methods. It will allow us to derive well-posedness results and asymptotic stability properties of the closed-loop systems. Some numerical simulations illustrate the convergence property of the solutions to the closed-loop nonlinear partial differential equations.

Timo Reis (University of Hamburg, Germany)

## Title:

Funnel control for the boundary-controlled heat equation

## Abstract:

The aim of tracking control is the design of a closed-loop controller such that the output of the system (approximately) follows a given reference signal. For a certain class of systems governed by ordinary differential equations, the "funnel-controller" suitably fulfills this job. We will introduce this controller and show that it can as well be applied to a heat equation with Neumann boundary control and output formed by the spatial integral of the Dirichlet boundary. The aim of tracking control is the design of a closed-loop controller such that the output of the system (approximately) follows a given reference signal. For a certain class of systems governed by ordinary differential equations, the "funnel-controller" suitably fulfills this job. We will introduce this controller and show that it can as well be applied to a heat equation with Neumann boundary control and output formed by the spatial integral of the design of systems governed by ordinary differential equations, the "funnel-controller" suitably fulfills this job. We will introduce this controller and show that it can as well be applied to a heat equation with Neumann boundary control and output formed by the spatial integral of the Dirichlet boundary.

## Author:

Felix Schwenninger (University of Hamburg)

## Title:

On integral input-to-state stability and equivalent notions for infinite-dimensional systems

## Abstract:

Notions like *input-to-state stability* and *integral input-to-state stability* are well-known in finitedimensional system theory. Recently, there has been growing interest in the study of these concepts for infinite-dimensional systems. The goal of this talk is to contribute towards this development. More precisely, we study the stability between the external input u and the state xof a linear system governed by the equation

$$\frac{d}{dt}x = Ax(t) + Bu(t); \quad x(0) = x_0,$$

where A and B are (typically unbounded) operators. Formally, stability notions can be interpreted as the boundedness of the input-to-state mapping

$$U \rightarrow X: x(\cdot) \rightarrow x(\tau), \quad \tau > 0,$$

where the topology (the norm) of the function space U is determined by the particular stability notion, e.g., classical  $L_p$ -admissibility refers to  $U=L_p$ . We show that *integral input-to-state stability* can indeed be understood in this sense rigorously, drawing a connection to admissibility with respect to *Orlicz spaces*. In particular, we focus on stability with respect to functions in  $L_{\infty}$ . Furthermore, we study the relation between the different notions and compare them with *(zeroclass)*  $L_p$ -admissibility.

This is joint work with B. Jacob (Wuppertal), R. Nabiullin (Wuppertal) and J.R. Partington (Leeds).

Yuan Wang (Florida Atlantic University, FL, USA)

#### Title:

On notions of input-to-output stability for systems with time-delays

#### Abstract:

We will discuss a notion of input-to-output stability (IOS), the output-Lagrange IOS property, and its Lyapunov characterizations for systems with time-delays. The output-Lagrange IOS property for delay-free systems (those that are not affected by time delays) has played critical roles in the Lyapunov theory for a variety of output stability properties. It is expected that the output-Lagrange IOS property will again play the same roles for delay systems. Furthermore, some new features arise in the decay estimations of the Lyapunov-Krasovskii functionals, an analogy of Lyapunov functions for delay-free systems. We will also provide some comparisons among the various notions on output stability for delay systems.

## Author:

Fabian Wirth (University of Passau)

#### Title:

Lyapunov theorems for infinite-dimensional systems

#### Abstract:

For large classes of infinite-dimensional systems a Lyapunov theory can be developed following closely the finite-dimensional model to obtain the expected results in the sense that no specific infinite-dimensional phenomena appear. One of the standard properties of Lyapunov functions in this approach is that Lyapunov functions are coercive in the fixed point, i.e. the Lyapunov function is lower bounded by a continuous positive definite function of the norm. While this property is automatic (at least locally) in finite dimensions, it is by no means natural to assume coercivity. Indeed, for *CO*-semigroups on Hilbert spaces it may very well happen that no coercive quadratic Lyapunov function exists. Similarly, Yoshizawa constructions of Lyapunov functions frequently do not result in coercive functions. In this talk we will show that the assumption of coercivity is not essential in the application of Lyapunov arguments. We will discuss Lyapunov theorems and converse Lyapunov theorems for families of abstract infinite-dimensional systems. Some special cases are discussed, such as parametrized families of time-varying systems, infinite-dimensional switched linear systems and systems with uncertain feedback.

Alexander Zuyev (Max-Planck-Institut Magdeburg, Germany)

#### Title:

Approximate controllability and steering problem for a class of hyperbolic systems with smooth controls

#### Abstract:

This talk focuses on the approximate steering problem for a class of hyperbolic distributed parameter systems with finite-dimensional controls. An approach for solving this problem is proposed by using exact solutions to the steering problem for reduced systems and the spillover condition. This approach also allows to estimate the reachable sets and to study the approximate controllability property. To satisfy the spillover condition, we exploit L2-optimal controls for a family of finite-dimensional subsystems. These controls are smooth and can be constructed explicitly for a class of oscillating systems with one-dimensional input. We illustrate this control design scheme by mechanical examples with elastic beams and plates.

## Author:

Hans Zwart (University of Twente, The Netherlands)

#### Title:

Stabilization of infinite dimensional port-Hamiltonian systems by non-linear dynamic boundary control

#### Abstract:

The conditions for existence of solutions and stability, asymptotic and exponential, of a large class of boundary controlled systems on a 1D spatial domain with non-linear dynamic boundary control are given. The non-linear boundary controller is considered to be passive, with non-linear potential energy function and damping matrix. It is shown that under very natural assumptions the solutions of the partial differential equation with the non-linear dynamic boundary conditions exist globally. Furthermore, when energy dissipation is present in the controller, then it globally asymptotically stabilizes the partial differential equation. The class of equations under study encompass a large class of physical distributed parameter systems with nonlinear actuation at their boundaries.

Joint work with Hector Ramirez and Yann Le Gorrec.