CONVERGENCE TO EQUILIBRIUM FOR SYSTEMS OF PARABOLIC EQUATIONS WITH A MATRIX POTENTIAL

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Abstract. On a bounded domain $\Omega \subseteq \mathbb{R}^d$, consider the coupled system of parabolic equations

$$\frac{d}{dt}\begin{pmatrix} u_1 \\ \vdots \\ u_N \end{pmatrix} = \begin{pmatrix} A_1 u_1 \\ \vdots \\ A_N u_N \end{pmatrix} + V \begin{pmatrix} u_1 \\ \vdots \\ u_N \end{pmatrix} \qquad (t \geq 0),$$

where $A_1, \dots, A_N$ are elliptic operators on Ω with Neumann boundary conditions and where $V : \Omega \to \mathbb{R}^{N \times N}$ is a matrix-valued potential. While the solutions to a single parabolic equation on a bounded domain are well-known to converge to an equilibrium as $t \to \infty$, the matrix potential V can for instance introduce the existence of periodic solutions to the equation.

In this talk, we discuss sufficient conditions for the solutions to the above system to converge as $t \to \infty$. We shall see that there is a close connection between this question and geometric properties of the potential V with respect to the ℓ^p-unit ball in $\mathbb{R}^N$ – more precisely speaking, we impose the assumption on V to be *p-dissipative*.

What makes our analysis interesting is the fact that completely different methods are required for the cases $p = 2$ and $p \neq 2$: in the first case, standard Hilbert space techniques can be used, while the case $p \neq 2$ requires more sophisticated methods from spectral theory, the geometry of Banach spaces and semigroup theory.

This talk is based on joint work the Alexander Dobrick (Christian-Albrechts-Universität zu Kiel).

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