THE SEMIGROUP AT INFINITY AND ITS APPLICATIONS TO COUPLED PARABOLIC EQUATIONS

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Abstract. Let $(T_t)_{t\in[0,\infty)}$ be a bounded semigroup of positive operators on some Banach lattice E and suppose that there is $t_0 > 0$ such that T_{t_0} is quasi-compact. Then T_t converges to a finite rank projection as $t \to \infty$ with respect to the operator norm by a theorem of H. Lotz [3].

What seems truly remarkable about this theorem in comparison to more well-known asymptotic results is, that the semigroup is not assumed to be strongly continuous. Therefore it allows to investigate norm convergence of semigroups lacking strong continuity; as they frequently occur in, e.g., stochastic applications. However, a main ingredient is the positivity of the semigroup. Recent spectral theoretic results from [1] and fruitful ideas from [2] allow to substitute positivity by geometric features of E; even more generally, it is possible to characterize norm convergence of abstract semigroups in terms of the properties of a topological group, the so-called *semigroup at infinity*.

As an application of these ideas, we prove that the solutions of coupled parabolic equations on $\mathbb{R}^d$ converge with respect to the operator norm under suitable assumptions on the coefficients and the coupling potential. This is joint work with Jochen Glück.

References

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