(Anti-)Maximum principles – an operator theoretic approach

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Extensive literature has been devoted to study the operators for which the maximum and/or the anti-maximum principle holds. Combining an idea of Takáč (1996), with those from the recent theory of eventually positive C_0-semigroups, we look at some necessary and sufficient conditions for uniform (anti-)maximum principles to hold in an abstract setting of Banach lattices.

More precisely, if $A : \text{dom}(A) \subseteq E \to E$ is a closed, densely defined, and real operator on a complex Banach lattice E (or, in particular, an L^p-space), then we consider the equation

$$(\lambda - A)u = f$$

for real numbers λ in the resolvent set of A. We ask whether $f \geq 0$ implies $u \geq 0$ for λ in a *right* neighbourhood of an eigenvalue. In this case, we say that the uniform maximum principle is satisfied. Analogously, when the implication $f \geq 0$ implies $u \leq 0$ holds for λ in a *left* neighbourhood of an eigenvalue, we say that the uniform anti-maximum principle holds.

We will also see how these abstract results can be applied to various concrete differential operators. In addition, we shall look at a characterization for both the individual maximum and anti-maximum principle to hold simultaneously. This is joint work with Jochen Glück.

References

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