Robust Graph Isomorphism, Quadratic Assignment, and VC dimension

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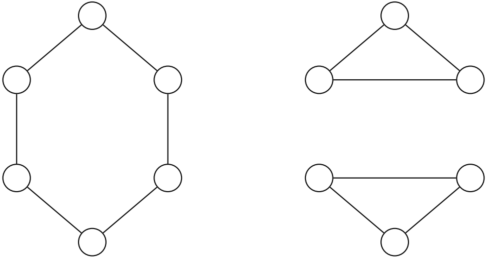
Graph edit distance



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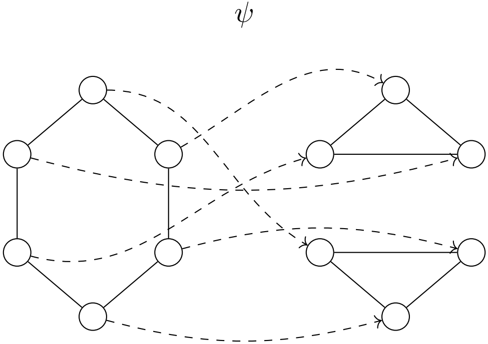

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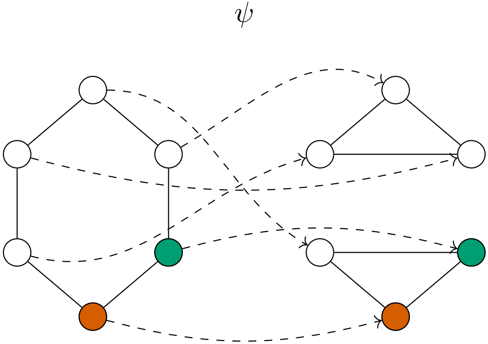

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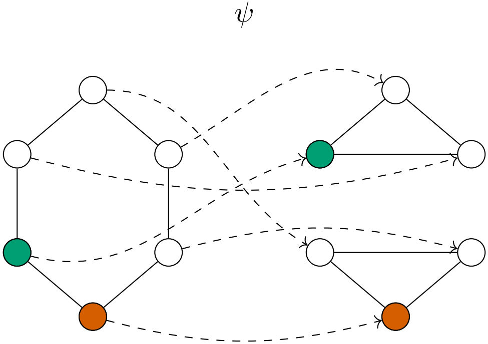

Graph edit distance

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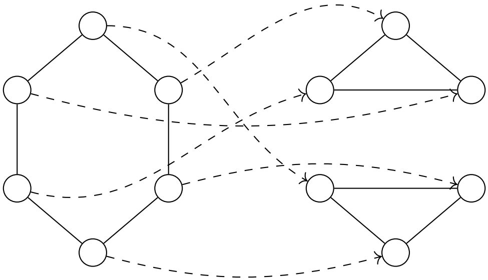

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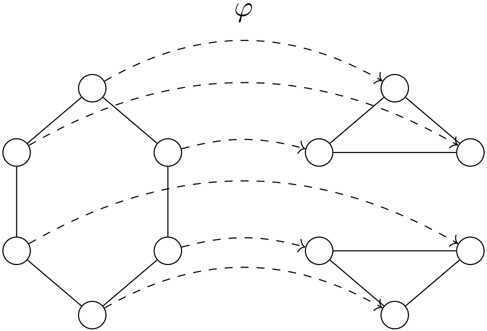
Graph edit distance



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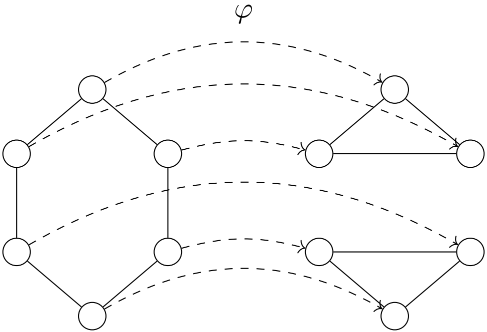

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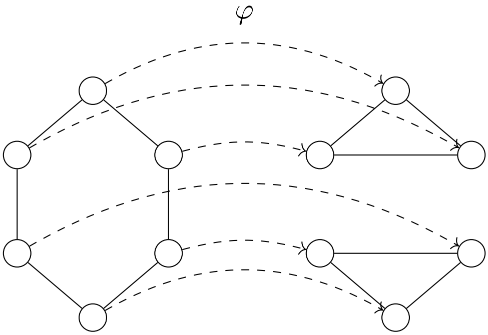

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$$\delta_{\text{edit}}(G, H) = \min_{\pi} \text{cost}(\pi) = 4$$

Graph edit distance

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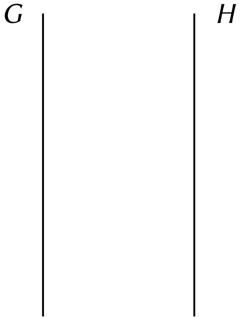
Theorem (Arora, Frieze, Kaplan, 2002)

Given G, H, ε, finding a bijection $\varphi : V(G) \to V(H)$ with $\operatorname{cost}(\varphi) \leq \delta_{\text{edit}}(G, H) + \varepsilon n^2$ can be carried out in time $n^{O(\log n/\varepsilon^2)}$.

GED using Arora-Frieze-Kaplan rounding procedure



Define *mixed associations*

$$M_\alpha(v, v') := \{(w, \alpha(w)) \mid E(v, w) \iff \neg E(v', \alpha(w))\}$$

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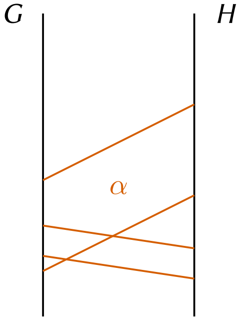

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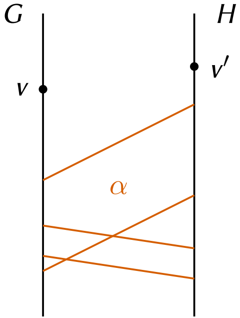

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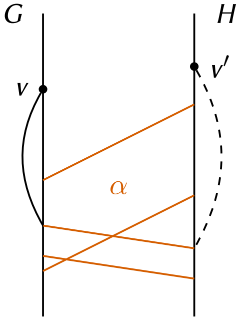

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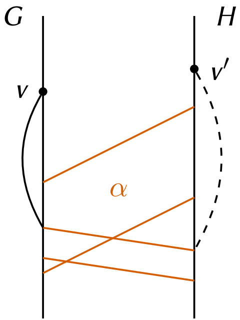

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Then one can show that

$$\mathrm{cost}(\varphi_{\alpha^*}) \leq \delta_{\mathrm{edit}}(G, H) + \varepsilon n^2.$$

ε-approximation

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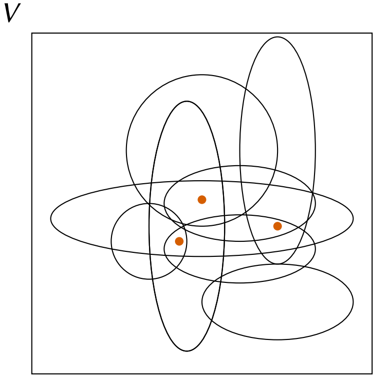
Lemma (Chazelle, 2000)

Let $\mathcal{H} \subseteq 2^V$ be any set system. For any $\varepsilon > 0$, there is an ε-approximation for $\mathcal{H}$ of size $O(\log n/\varepsilon^2)$.

VC dimension

Suppose a set system $\mathcal{H} \subseteq 2^V$. We call $X \subseteq V$ *shattered*, if for any subset $Y \subseteq X$ there is $H \in \mathcal{H}$ such that $X \cap H = Y$.



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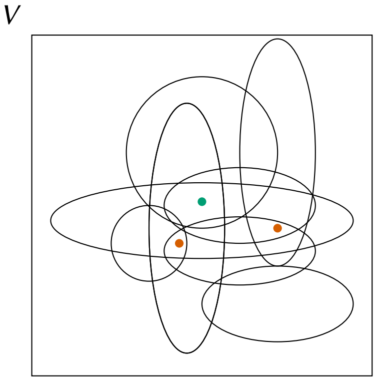

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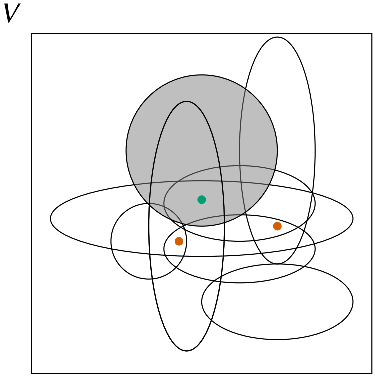

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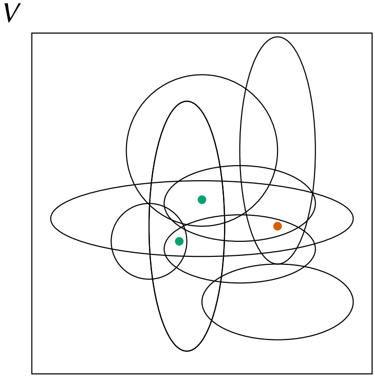

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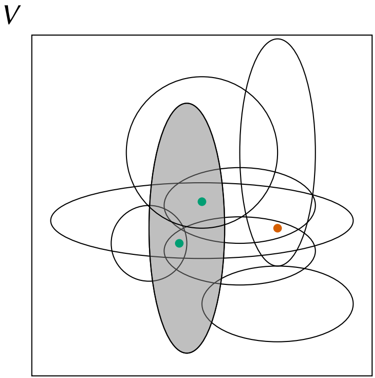

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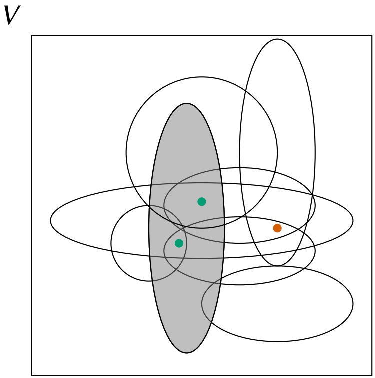

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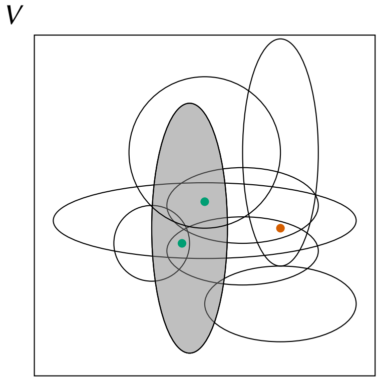

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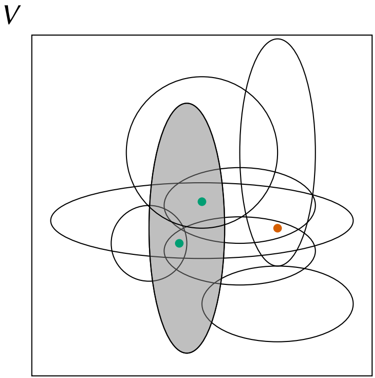

$d \leq \log n$

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Lemma

If G, H have VC-dim *at most d, then $\mathcal{M}_{\varphi^*}$ has* VC-dim *at most $10d$.*

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If G, H have VC-dim *at most d, then $\mathcal{M}_{\varphi^*}$ has* VC-dim *at most $10d$.*

Lemma (Li, Long, Srinivasan, 2001)

If a set system $\mathcal{H}$ has VC-dim *at most d, then for any $\varepsilon > 0$, there is an ε-approximation for $\mathcal{H}$ of size $O(d/\varepsilon^2)$.*

First result

Hence, if G, H, have VC-dim at most d, then we can do the following:

- Show that there is an α^* of size $O\left(\log n / \varepsilon^2\right)$ so that for all v, v',

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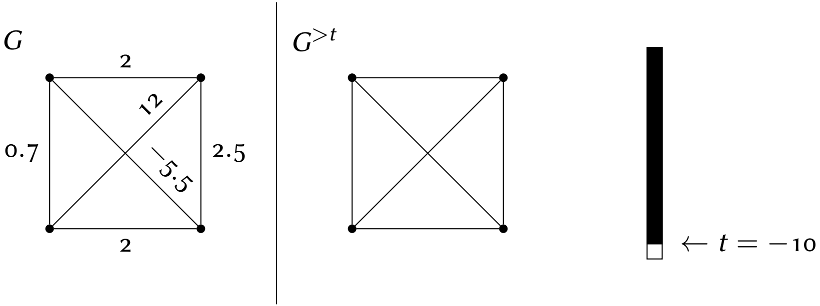

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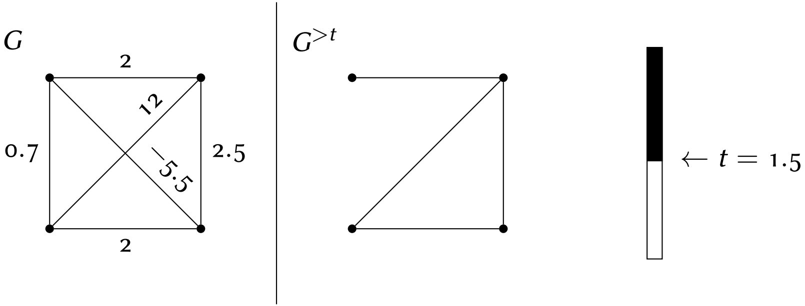

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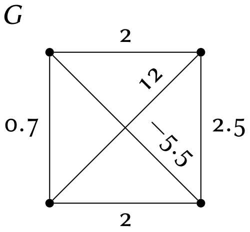

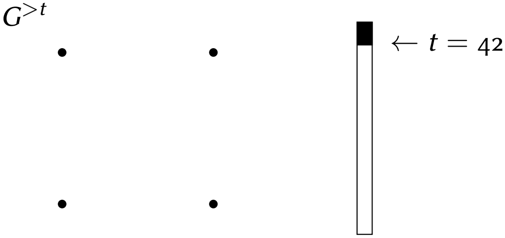

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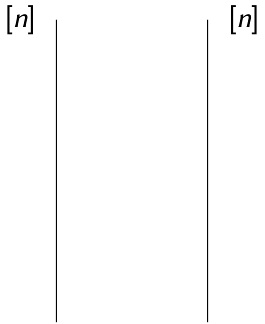
Quadratic Assignment Problems of bounded VC dimension



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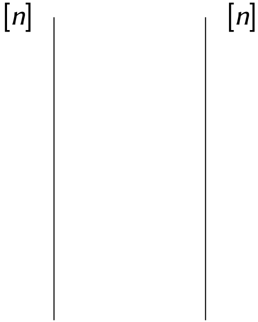

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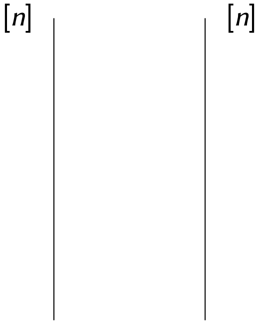

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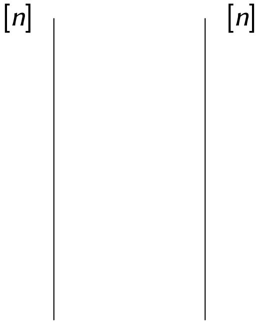

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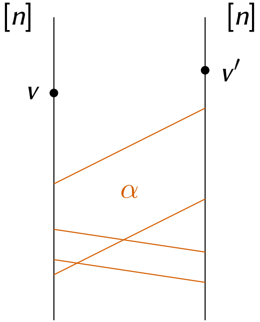

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$$M_\alpha^t(v, v') := \{(w, \alpha(w)) \mid \quad c(v, v', w, \alpha(w)) > t \quad \}$$

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- $n^{O(d/\varepsilon^2)}$ on graphs of VC-dim $\leq d$
- $n^{O_\varepsilon(d)}$ on arbitrary QAPs of VC-dim $\leq d$

Robust Graph Isomorphism

- Input: graphs G, H of size n such that either $G \cong H$ or $\delta_{\text{edit}}(G, H) > \varepsilon n^2$.
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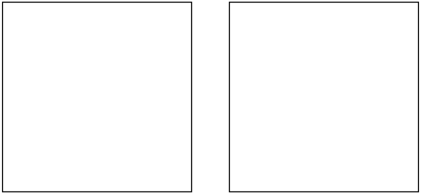
Question. Can we solve this problem only using combinatorial techniques?

Weisfeiler-Leman algorithm

If G, H are k-WL equivalent, then for each $k-1$ tuple $\bar{\mathbf{v}}$ in G there is $k-1$ tuple $\bar{\mathbf{w}}$ in H such that



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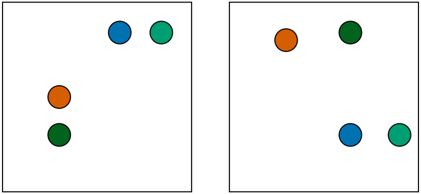

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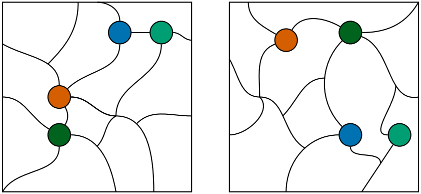

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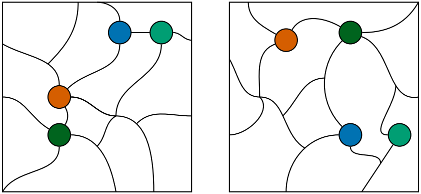
leaves us with same colour partition on both sides.



Second main result

Theorem

Suppose VC-dim$(G) \leq d$. *If G and H are* $O(d/\varepsilon^2 \log 1/\varepsilon)$*-WL equivalent, then* $\delta_{\text{edit}}(G, H) \leq \varepsilon n^2$.

Second main result

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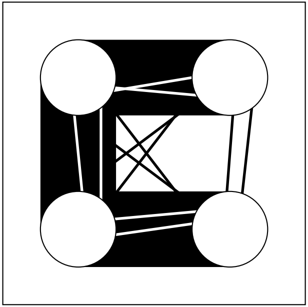
Suppose VC-dim$(G) \leq d$. *If G and H are $O(d/\varepsilon^2 \log 1/\varepsilon)$-WL equivalent, then $\delta_{\text{edit}}(G, H) \leq \varepsilon n^2$.*

We can show that there is a tuple in G of such size, such that individualising it and running colour refinement:



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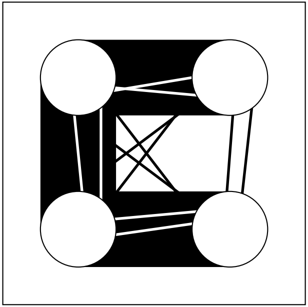

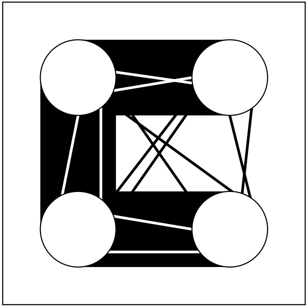

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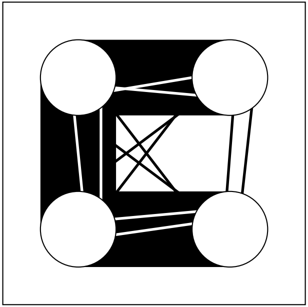

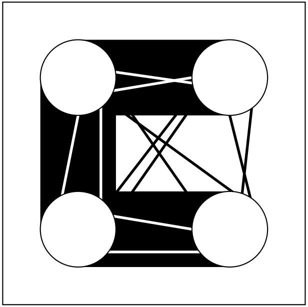

Results

Approximating edit distance

- $n^{O(\log n/\varepsilon^2)}$
- $n^{O(d/\varepsilon^2)}$ on graphs of VC-dim $\leq d$
- $n^{O_\varepsilon(d)}$ on arbitrary QAPs of VC-dim $\leq d$

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Robust GI lower bounds

From known constructions (CFI graphs), we can conclude the following lower bounds.

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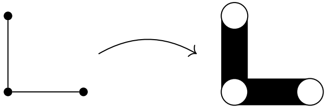

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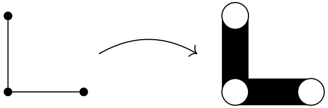

Edit distance grows quadratically. [Pikhurko]

Results

Approximating edit distance

- $n^{O(\log n/\varepsilon^2)}$
- $n^{O(d/\varepsilon^2)}$ on graphs of VC-dim $\leq d$
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Robust GI using WL

- $O(\log n/\varepsilon^2)$-WL
- $O(d/\varepsilon^2 \cdot \log(1/\varepsilon))$-WL on graphs of VC-dim $\leq d$

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Approximating edit distance

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Robust GI using WL

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- $O(d/\varepsilon^2 \cdot \log(1/\varepsilon))$-WL on graphs of VC-dim $\leq d$
- $\Theta(\text{poly}(\varepsilon))$-WL on graphs bounded VC-dim / bounded colour class size