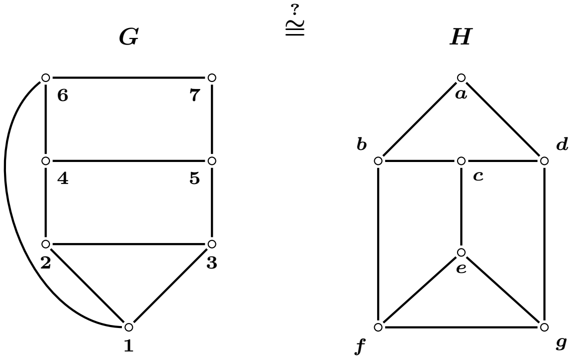
Dynamic Planar Graph Isomorphism is in DynFO

Samir Datta Asif Khan Felix Tschirbs
Nils Vortmeier Thomas Zeume

AlMoTh, 3 March 2026

The Planar Graph Isomorphism Problem



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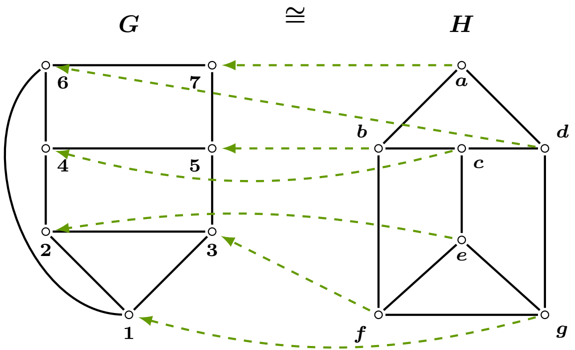

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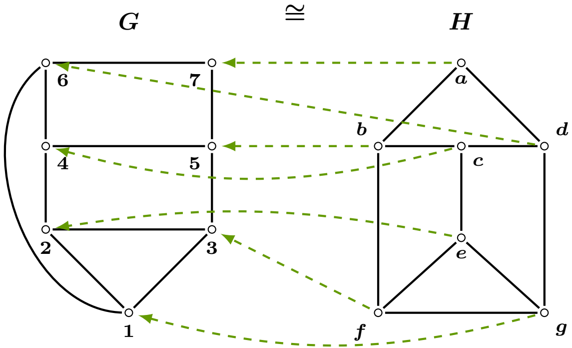
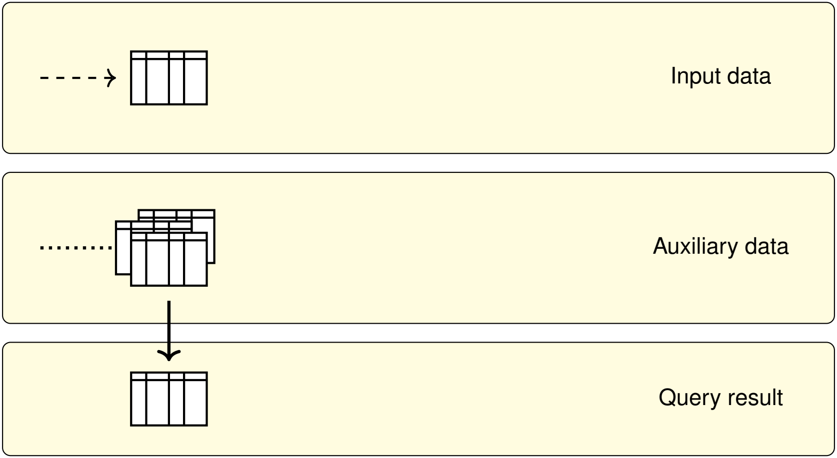


Isomorphism of planar graphs can be solved

- in linear time [Hopcroft and Wong 1974]
- with logarithmic space [Datta et al. 2022]

Setting of Dynamic Query Evaluation



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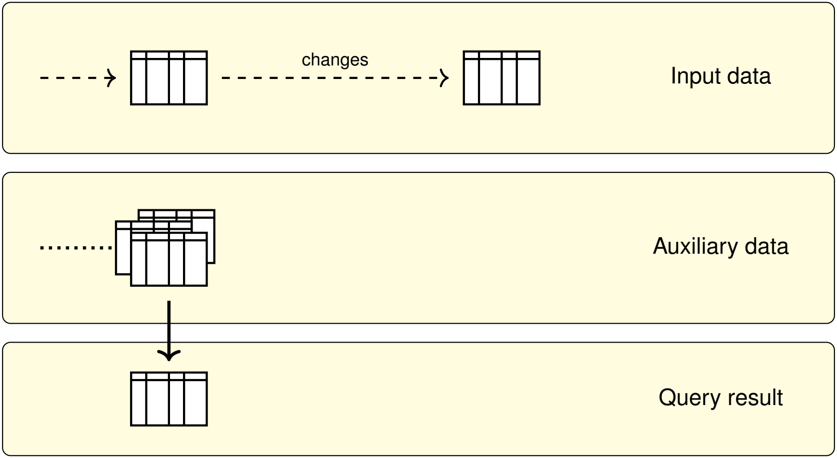

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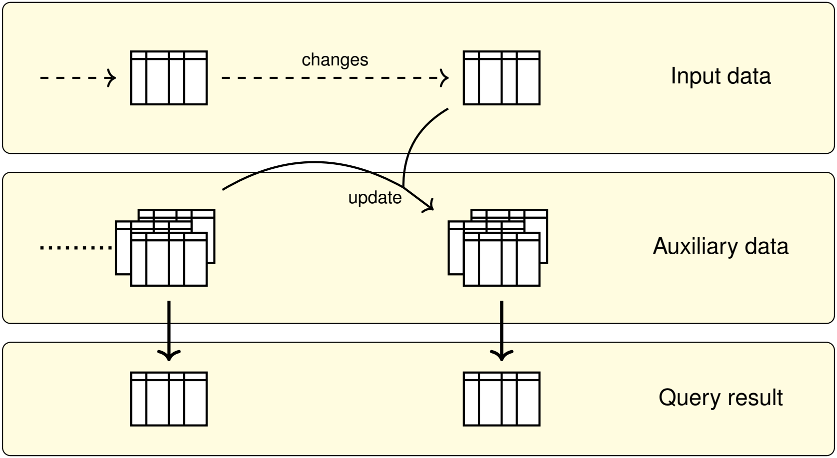

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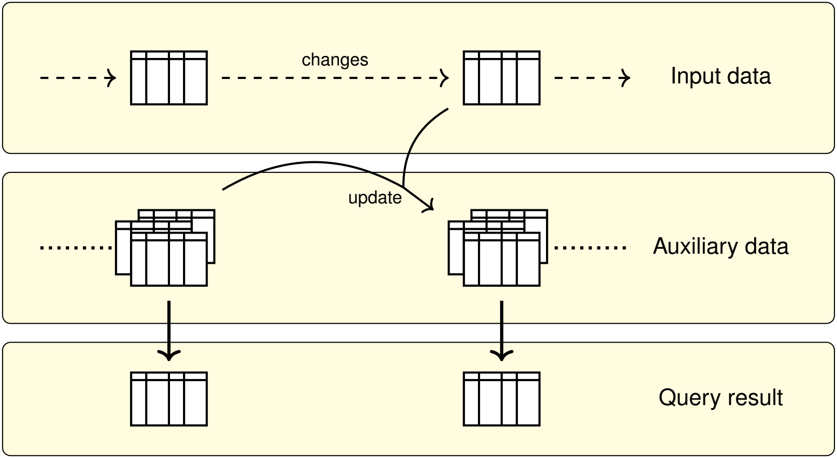

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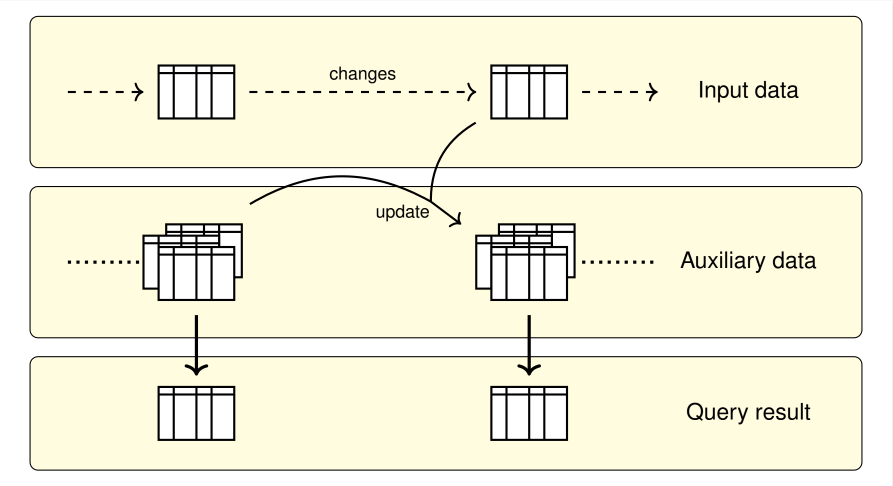
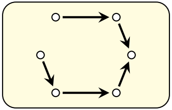


- Descriptive approach: express update using logical formulas
  - input and auxiliary data is relational

Example: Reachability for acyclic graphs [Patnaik and Immerman 1997]

G



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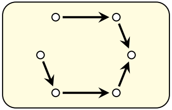

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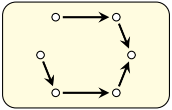

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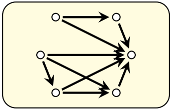

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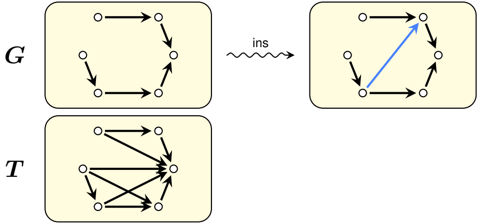

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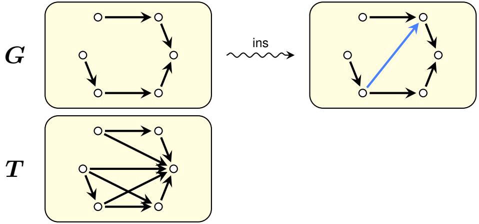

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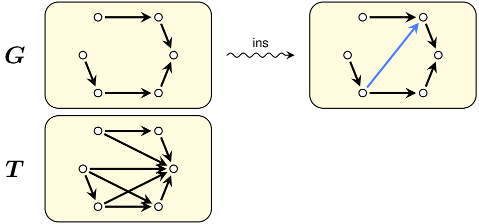

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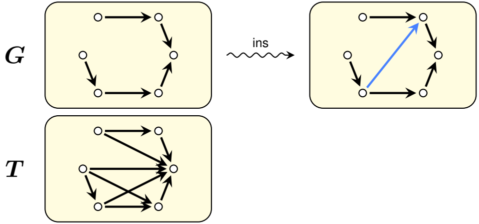

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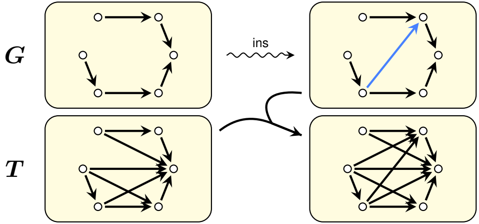

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- Update formula for T after edge insertion:

$$\phi^T_{\text{ins}}(x, y; u, v) = T(x, y) \vee \big(T(x, u) \wedge T(v, y)\big)$$

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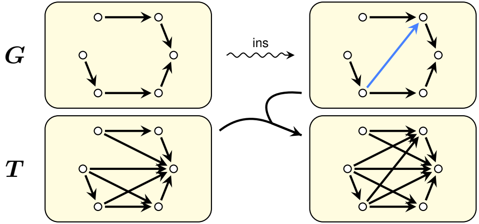

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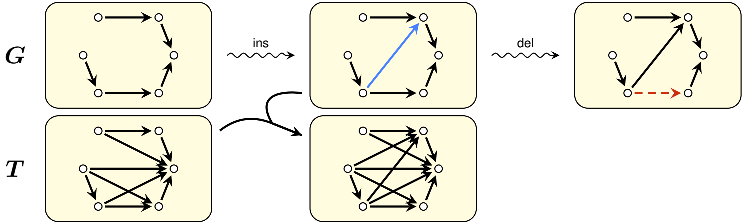

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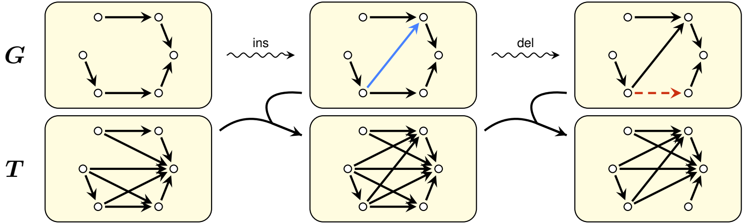

- Update formula for T after edge deletion:

$$\phi^T_{\text{del}}(x, y; u, v) = T(x, y) \wedge \Big[\neg\big(T(x, u) \wedge T(v, y)\big) \vee \exists z \exists z' \big((z \neq u \vee z' \neq v) \wedge T(x, z) \wedge E(z, z') \wedge T(z', y) \wedge T(z, u) \wedge \neg T(z', u)\big)\Big]$$

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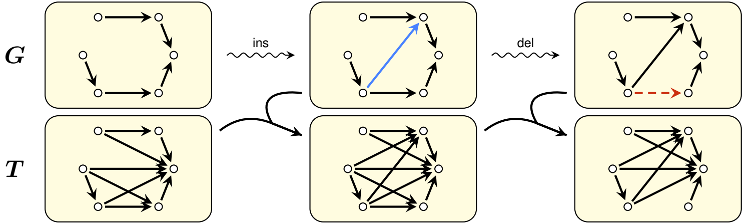

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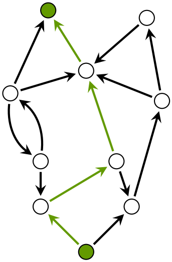
DynFO

Queries maintained using first-order update formulas

Prior DynFO maintainability results

Reachability is in **DynFO** for

- acyclic graphs
  [Dong and Su 1995, Patnaik and Immerman 1997]
- undirected graphs
  [Patnaik and Immerman 1997, Dong and Su 1998, Grädel and Siebertz 2012]
- directed graphs
  [Datta, Kulkarni, Mukherjee, Schwentick, Zeume 2015]



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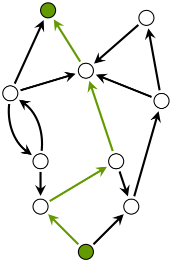
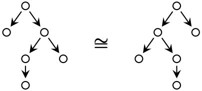


Graph queries in **DynFO** that are relevant for this talk:

- Tree isomorphism
  [Etessami 1998]



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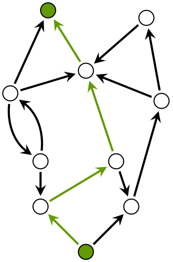

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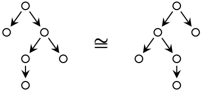
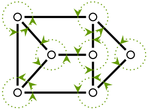
- Graph planarity + maintenance of plane embedding
  [Datta, Khan, Mukherjee 2023]





Main result

Theorem

The dynamic graph isomorphism problem for planar graphs is in **DynFO**

- Allowed changes: inserting and deleting single edges
- The graph has to stay planar the whole time

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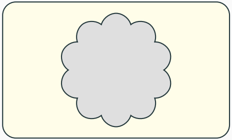
Theorem, rephrased

The dynamic graph isomorphism problem for planar graphs can be maintained by

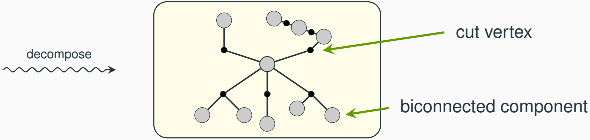
- $\mathbf{AC^0}$-circuits
- a dynamic constant-time parallel algorithm

with auxiliary data of polynomial size

Proof overview

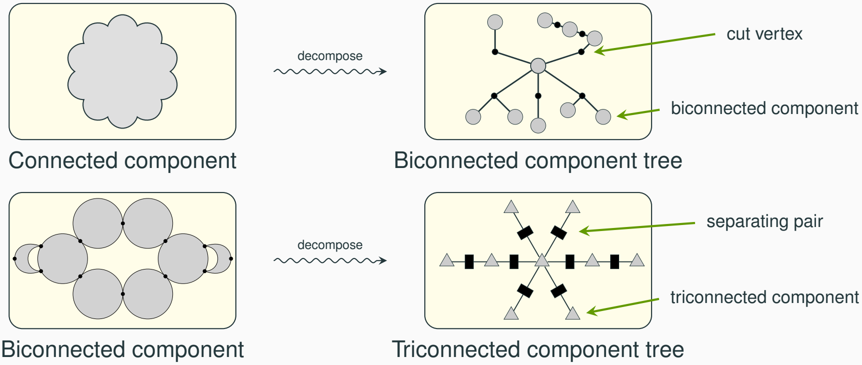


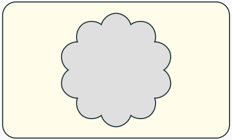
Connected component



Biconnected component tree

Proof overview





Connected component

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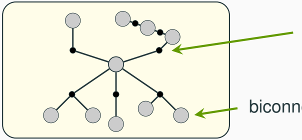

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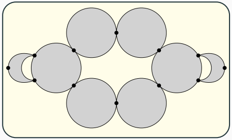

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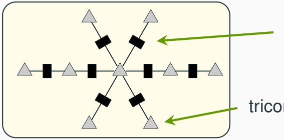

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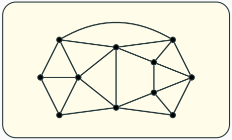

Triconnected component

solve

determine isomorphism via canonical embedding

Effects of edge changes on the component trees

- Difficulty: biconnected and triconnected component trees may change drastically after an edge change

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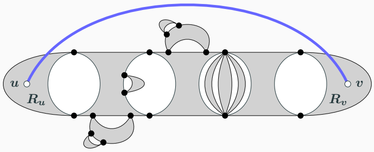

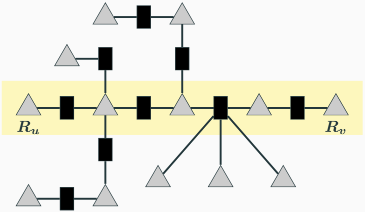

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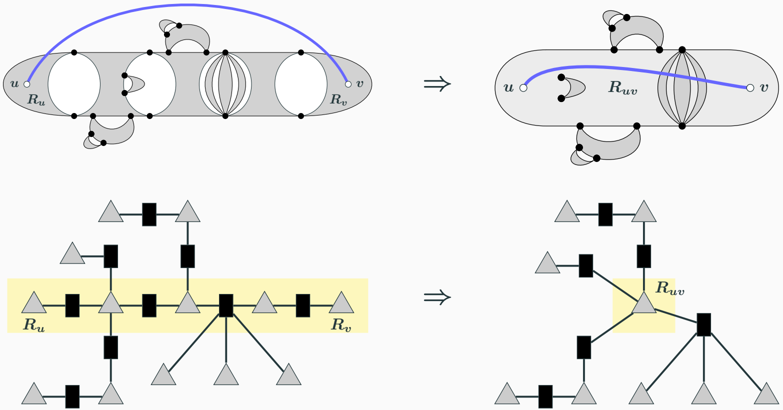

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3. Maintaining isomorphism of the component trees

   ▷ Generalizing Etessami's approach for tree isomorphism [Etessami 1998]

Maintaining isomorphism of 3-connected components

Embedding 3-connected planar graphs

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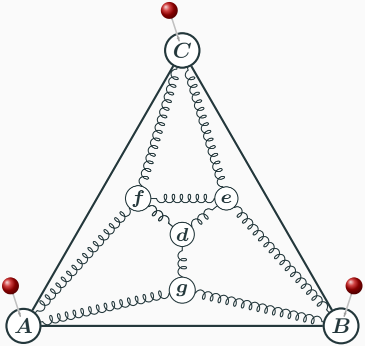

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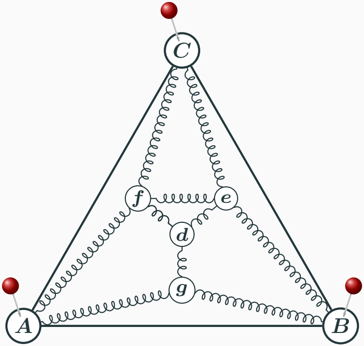

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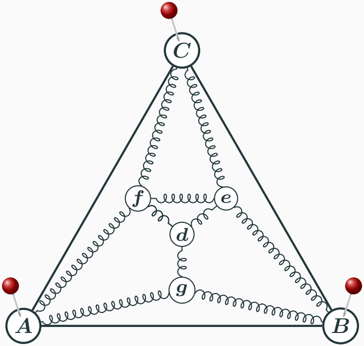

- Idea of our approach:
  1. Maintain the spring embedding for each possible choice of pinned vertices
  2. Determine whether $\mathbf{3}$-connected components are isomorphic by checking whether their spring embeddings are equal (for some choice of pinned vertices)

Maintaining the spring embedding

- Fixing vertices A, B, C to $(0,0), (0,1), (1,0)$ gives as equations for the x-coordinates

$$\begin{aligned} \deg(v) x_v &= \sum_{(v,w) \in E} x_w && \forall v \notin \{A, B, C\} \\ x_A &= 0 \\ x_B &= 0 \\ x_C &= 1 \end{aligned}$$

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- Goal: maintain the inverse $\mathbf{T}^{-1}$
  - then obtain the embedding via $\mathbf{x} = \mathbf{T}^{-1}\,\mathbf{b}$

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  - ▷ For small changes, the Sherman-Morrison-Woodbury (SMW) formula can be used [Henderson and Searle 1981]

$$(\mathbf{T}+\mathbf{U}\mathbf{V}^{\mathsf{T}})^{-1} = \mathbf{T}^{-1} - \mathbf{T}^{-1}\mathbf{U}(\mathbf{I}+\mathbf{V}^{\mathsf{T}}\mathbf{T}^{-1}\mathbf{U})^{-1}\mathbf{V}^{\mathsf{T}}\mathbf{T}^{-1}$$

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  - ▷ For small changes, the Sherman-Morrison-Woodbury (SMW) formula can be used [Henderson and Searle 1981]

    $$(\mathbf{T} + \mathbf{U}\mathbf{V}^{\mathsf{T}})^{-1} = \mathbf{T}^{-1} - \mathbf{T}^{-1}\mathbf{U}(\mathbf{I} + \mathbf{V}^{\mathsf{T}}\mathbf{T}^{-1}\mathbf{U})^{-1}\mathbf{V}^{\mathsf{T}}\mathbf{T}^{-1}$$
- Many $\mathbf{3}$-connected components may merge after an edge insertion!

Challenges for maintaining the matrix inverse

- The entries of the matrix inverse get very large!
  - ▷ We maintain the entries modulo small primes
- Matrix inverses do not exist modulo some small primes!
  - ▷ We maintain the entries modulo more small primes ☺
  - ▷ …and periodically recompute the information [Datta, Mukherjee, Schwentick, V., Zeume 2019]
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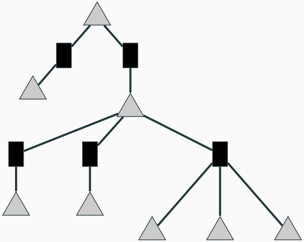
- Many $\mathbf{3}$-connected components may merge after an edge insertion!
  - ▷ We maintain the inverses also for all $\mathbf{3}$-connected components that are the result of an edge insertion
    - ⋆ Then, in all cases only a constant number of matrices need to be combined
  - ▷ We maintain the result of some terms of the SMW formula

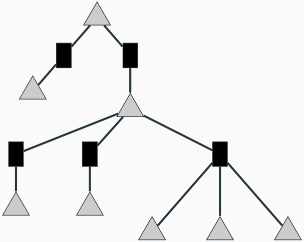
Maintaining isomorphism of 2-connected components

Auxiliary data for 2-connected components

- To maintain: Given two triconnected component trees, are the described graphs isomorphic?



$\stackrel{?}{\cong}$



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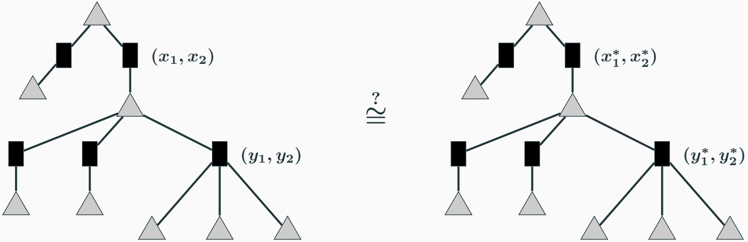

- We maintain implications of the following form as auxiliary data
  - If the graph of the subtree of (y_1, y_2) is isomorphic to the graph of the subtree of (y_1^*, y_2^*)
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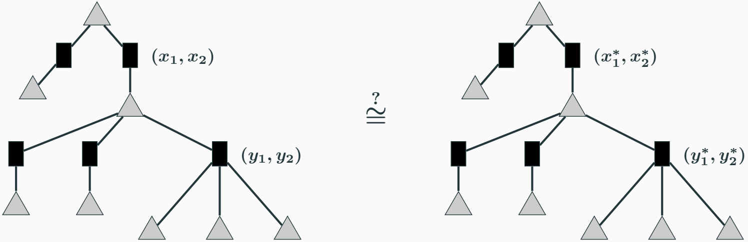

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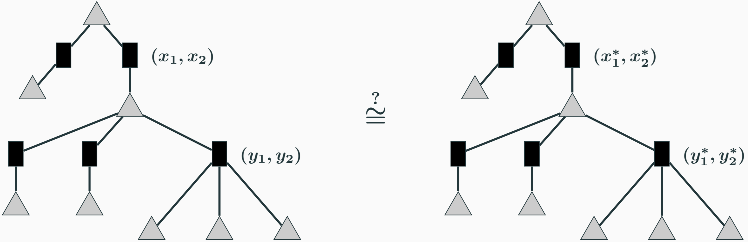

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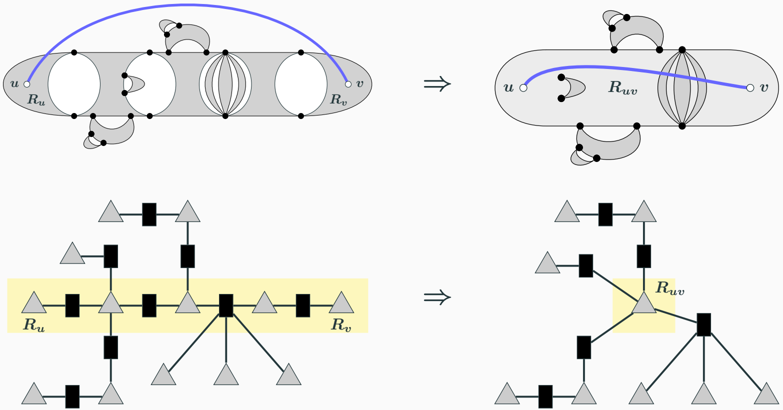
- . . . and similar facts for triconnected component nodes
- . . . and some more technical auxiliary data

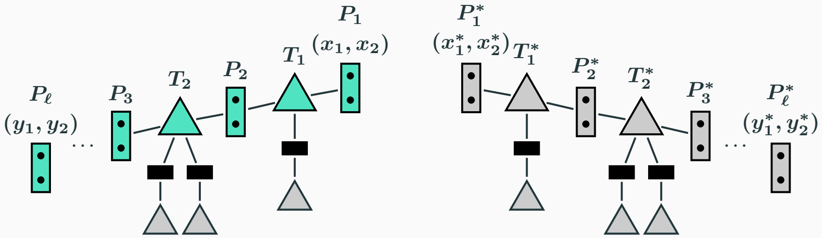
Recall: Effects of edge changes on the component trees

- Triconnected component trees may change drastically after an edge change



Maintenance of the auxiliary data: a central case

- Checks for updating the auxiliary data when a $\mathbf{3}$-connected component collapses:



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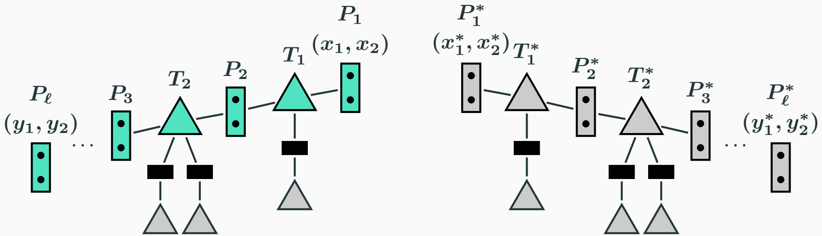

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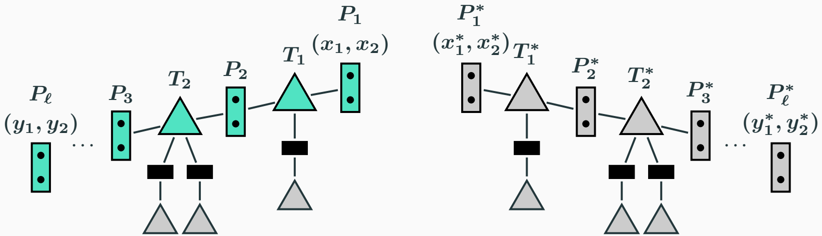

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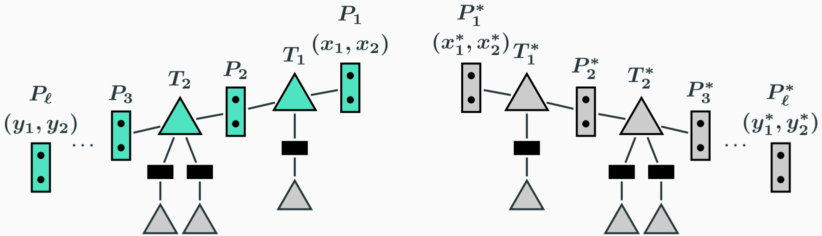

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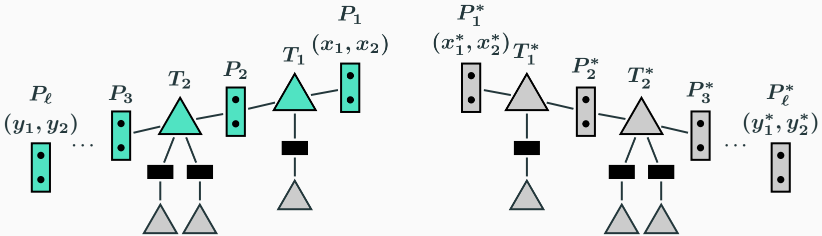

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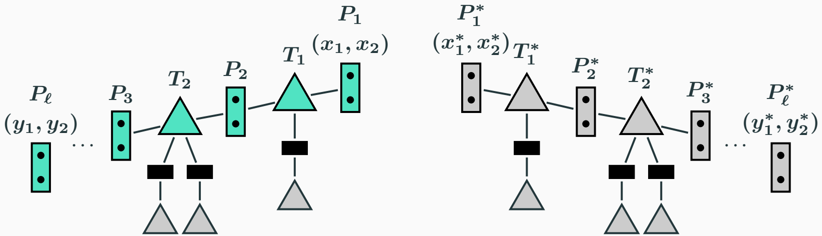

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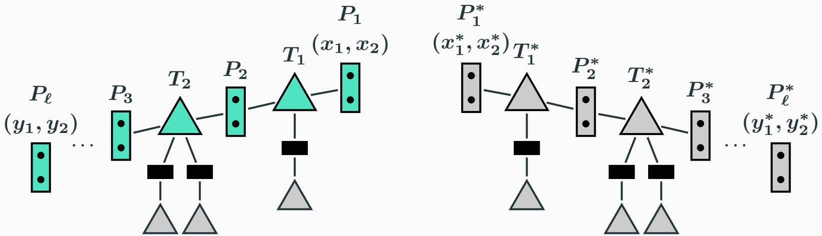

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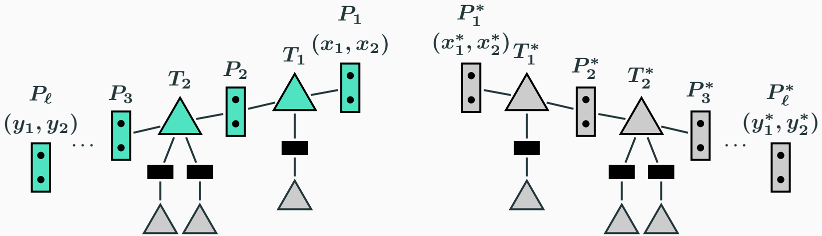

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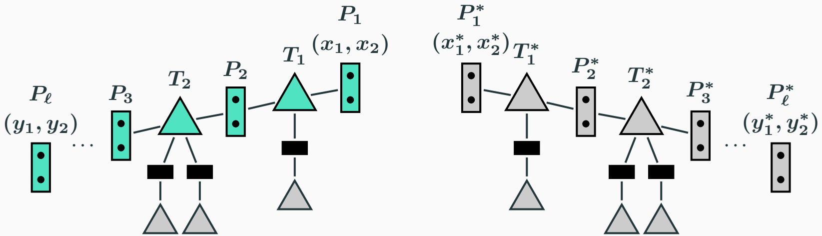

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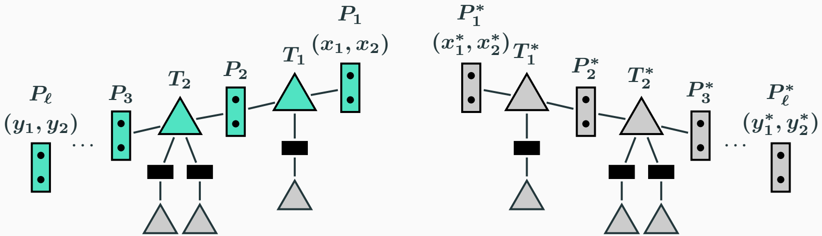

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    - check in auxiliary data whether the subtrees of the separating pairs are isomorphic
- Additional data that we need for this:
  - distances on the path
  - embedding information to consistently map vertices from P_i to P_i^*

Conclusion

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  - rephrased: . . . using a parallel constant-time algorithm with polynomial auxiliary data

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  - Whether two planar graphs are isomorphic can be maintained using first-order update formulas
  - rephrased: …using a parallel constant-time algorithm with polynomial auxiliary data
- We only had a brief glimpse of the proof
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  - not mentioned at all: from isomorphism of biconnected components to isomorphism of connected components
- So far: we can solve the decision problem
  - Next step: maintaining a witnessing isomorphism

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