

Advertisement: Bachelor's Thesis

Topic: Hamiltonicity of expander graphs

Supervisor: Prof. Dr. Stefan Glock

Description: A *Hamilton cycle* in a graph G is a cycle that contains every vertex of G . The study of Hamiltonicity of graphs is one of the main topics of graph theory and connects to many other areas such as coding theory, optimisation and theoretical computer science. A graph G is called a C -*expander* if $|N(X)| \geq C|X|$ for every $X \subseteq V(G)$ with $|X| < n/2C$, where $N(X)$ denotes the neighbourhood of X in G . Recently, Draganić, Montgomery, Munhá Correia, Pokrovskiy and Sudakov proved that there is an absolute constant $C > 0$ such that every C -expander has a Hamilton cycle. This implies in a strong form a well-known conjecture of Krivelevich and Sudakov from 2003.

Thesis goals: The candidate is expected to study in-depth the research paper “Hamiltonicity of expanders: optimal bounds and applications” by Draganić et al. and write a detailed exposition on it. The final thesis should in particular introduce the research area (Hamiltonicity / expander graphs), survey the history and significance of the research problem, and explain the relevant methods and their advantages/limitations. Further potential ways to make original scientific contributions are to seek simplifications in the proof arguments, improve the exposition where possible, strengthen or generalize the main result (e.g.: value of the constant C , cycle factors), or adapt the methods to obtain similar results in other settings.

Main article:

- N. Draganić, R. Montgomery, D. Munhá Correia, A. Pokrovskiy and B. Sudakov, Hamiltonicity of expanders: optimal bounds and applications, preprint (2024).

🔗 <https://arxiv.org/pdf/2402.06603>

Further reading:

- S. Glock, D. Munhá Correia and B. Sudakov, Hamilton cycles in pseudorandom graphs, *Advances in Mathematics* 458 (2024), 109984.
- D. Hefetz, M. Krivelevich and T. Szabó, Hamilton cycles in highly connected and expanding graphs, *Combinatorica* 29 (2009), 547–568.
- M. Krivelevich and B. Sudakov, Sparse pseudo-random graphs are Hamiltonian, *Journal of Graph Theory* 42 (2003), 17–33.