

Advertisement: Master's Thesis

Topic: The existence conjecture on combinatorial designs

Supervisor: Prof. Dr. Stefan Glock

Description: Let V be a set of size n . An (n, q, r) -Steiner system is a collection $\mathcal{S}$ of subsets of V of size q , with the property that every subset of V of size r is contained in precisely one set in $\mathcal{S}$. In 1853, the Swiss geometer Jakob Steiner asked for which parameters n, q, r such objects exist. One can observe some obviously necessary divisibility conditions. The existence conjecture speculated that these necessary conditions are, when q and r are fixed and n is sufficiently large, also sufficient. This was finally proved in a breakthrough by Peter Keevash announced in 2014.

Thesis goals: The candidate is expected to study in-depth the research paper “A short proof of the existence of designs” by Keevash and write a detailed exposition on it. The final thesis should in particular introduce the research area (combinatorial designs / hypergraph decompositions), survey the history and significance of the research problem, and explain the relevant methods and their advantages / limitations (Rödl nibble / random greedy process, absorption). Further potential ways to make original scientific contributions are to seek simplifications in the proof arguments, improve the exposition where possible, strengthen or generalize the main result (e.g.: formally verify that the constructed absorbers are “refined”), or adapt the methods to obtain similar results in other settings (e.g.: decompositions into arbitrary hypergraphs, not just cliques).

Main article:

- P. Keevash, A short proof of the existence of designs, preprint (2024).
 <https://arxiv.org/pdf/2411.18291>

Further reading:

- M. Delcourt and L. Postle, Refined absorption: a new proof of the existence conjecture, preprint (2024).
- S. Glock, D. Kühn, A. Lo and D. Osthus, The existence of designs via iterative absorption: hypergraph F -designs for arbitrary F , *Memoirs of the American Mathematical Society* 284 (2023), monograph 1406.
- P. Keevash, The existence of designs, preprint (2014).