Frequency detection with orthogonal polynomials on the bitorus

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In this presentation, we consider the problem of recovering the frequency pairs of a real-valued bivariate exponential sum from sample values. The mathematical problem is as follows. Given function values

$$f_{k,l} := \sum_{j=1}^{M} \rho_j \mathrm{e}^{\mathrm{i}(k\lambda_j + l\mu_j)}, \quad k = 0, ..., N_1 - 1,\ l = 0, ..., N_2 - 1$$

with complex coefficients $\rho_j \neq 0$ and $(\lambda_j, \mu_j) \in (0, \pi)^2$ we want to find good estimates of the integer M and the frequency pairs (λ_j, μ_j).
For the one-dimensional case, there are many methods to solve this problem e.g Prony's method, Szegő's method, the Matrix-Pencil method and several improvements of the above methods. Since the Szegő-method has proven to be very stable under the influence of noise, we extend this method to the two-dimensional case.