Wavelet Approximation Theory in Higher Dimensions

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Abstract

The success of wavelets in applications such as signal and image processing is to a large degree based on their approximation theoretic properties. The central mathematical statement that summarizes these properties is the characterization of smoothness spaces such as Besov spaces in terms of summability conditions on the wavelet coefficients. In more modern terminology, the elements of a Besov space can be understood as the sparse signals with respect to the wavelet systems.

In higher dimensions, an increasing variety of generalized wavelet systems becomes available, each of which inducing its own spaces of sparse signals. In this talk, the methodology of *coorbit spaces*, developed by Feichtinger and Gröchenig, is introduced as a very general and flexible means of describing spaces of sparse signals over generalized wavelet systems. The method depends crucially on Wiener amalgam estimates, and it provides very general consistency and discretization results for the wavelet systems under review. Here consistency refers to the important property that, for any two analysing wavelets within a well-understood class of functions, the spaces of sparse signals coincide.

Thus wavelet coorbit space theory provides a comprehensive and unified scheme that comprises many known examples such as isotropic wavelets and Besov spaces in higher dimensions, shearlets and their coorbit spaces in arbitrary dimensions, and many more.