Irregular Sampling and Wiener Amalgam Spaces

Hans G. Feichtinger

Abstract

The so-called irregular sampling theorem is a good example if one wants to illustrate the use of function spaces, Banach frames, Riesz projection bases etc. in action, and not just in an abstract Hilbert space setting.

The starting point is usually the famous Shannon sampling theorem, which allows to show that a band-limited signal, whose spectrum is supported on some interval of the frequency domain, can be recovered from equidistant samples, as long as they gap size is at most the inverse of the length of the spectral interval, i.e. if samples are taken at the Nyquist rate or at a higher sampling rate. Variants of this theorem allow to formulate this results, which in its original form is just an orthogonal expansion of a band-limited functions, in the context of weighted LpLp-spaces. The theory of irregular sampling, established in the early 90th, demonstrates that recovery is possible, usually in an iterative way, if only the maximal gap size is strictly less than the Nyquist rate.

For the proof it is very helpful to make use of so-called Wiener amalgam spaces, i.e. spaces which allow to control local and global properties of a function or a measure kind of independently. The talk will give hints, how such spaces can be easily understood and put to good use in order to settle the problem.

Although historically coorbit theory was established before the first generation of irregular sampling algorithms have been established one can motivate the atomic decomposition results that have been derived in the context of coorbit theory as irregular sampling results for the CWT or STFT (continuous wavelet transform or short-time Fourier Transform). Hence, in this context one can obtain results concerning regular and irregular Gabor families.