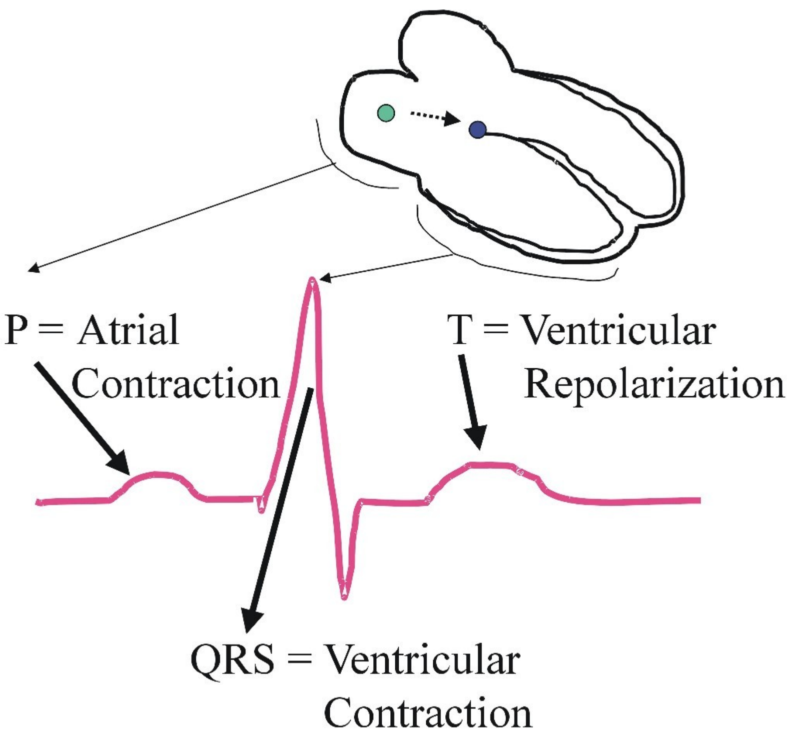
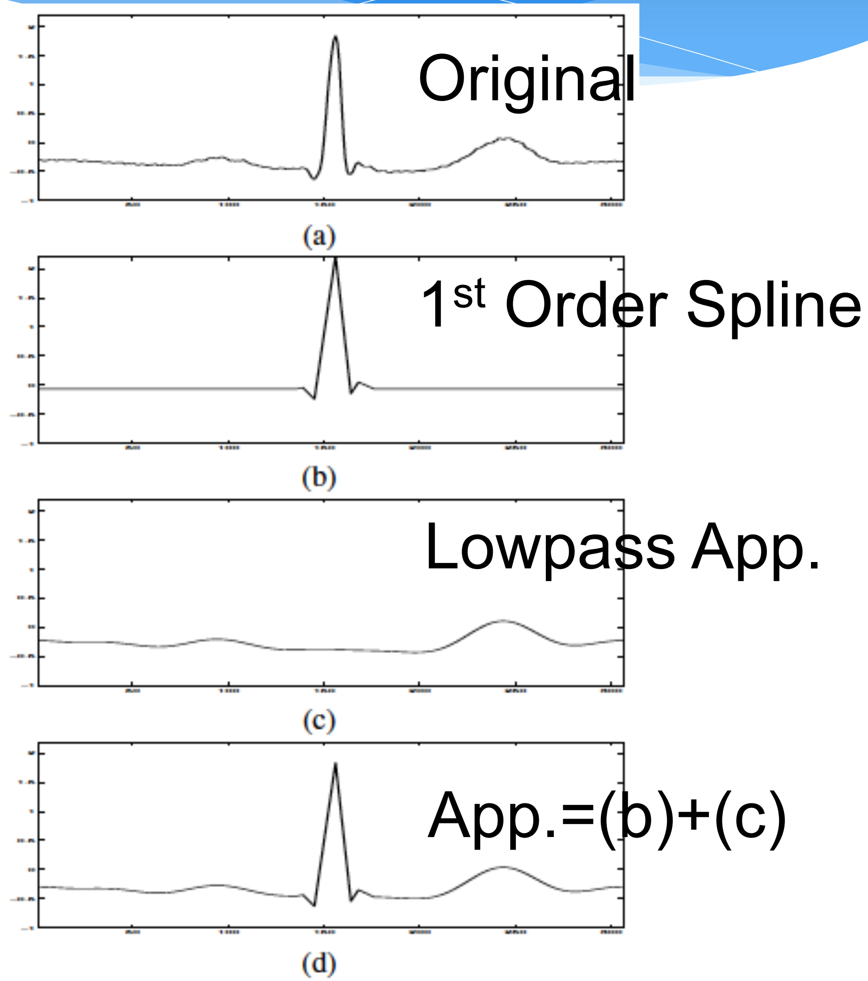
Outline

- Introduction of new class of signals
  - As an extension of bandlimited signals
- Sampling and Reconstruction
  - Noiseless case
  - Noisy case
- Application
  - Compression of ECG signals
  - Line-edge extraction

Compression of ECG Signals

Hao et al. (2005)



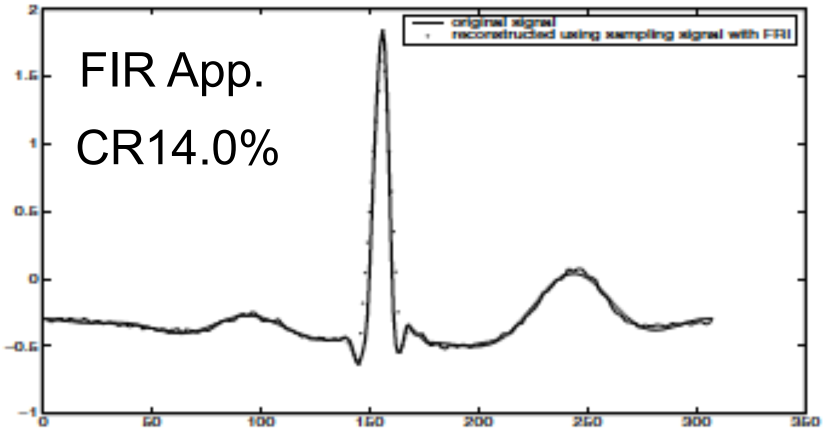


Approximation Results

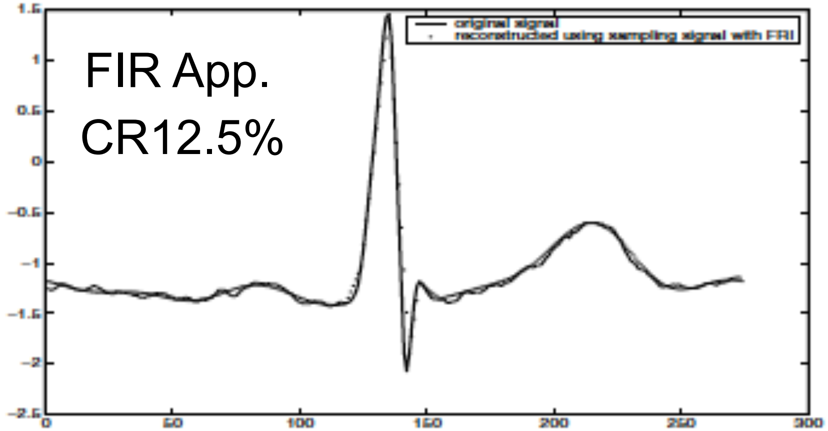
MIT/BIH Arrhythmia Database

Data 103

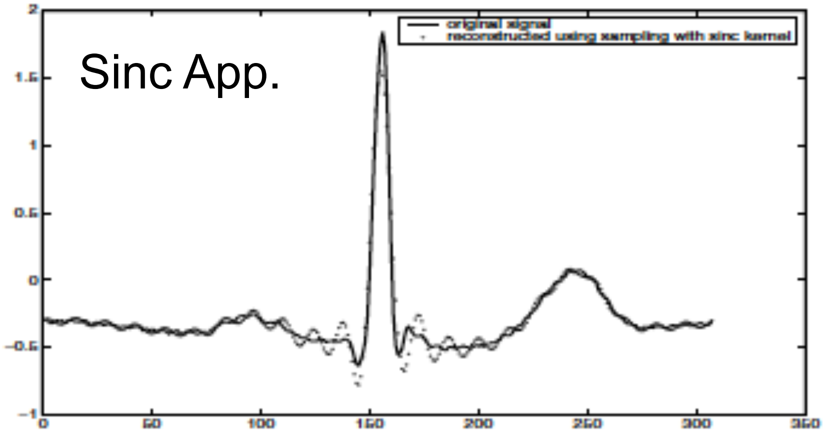
Data 116



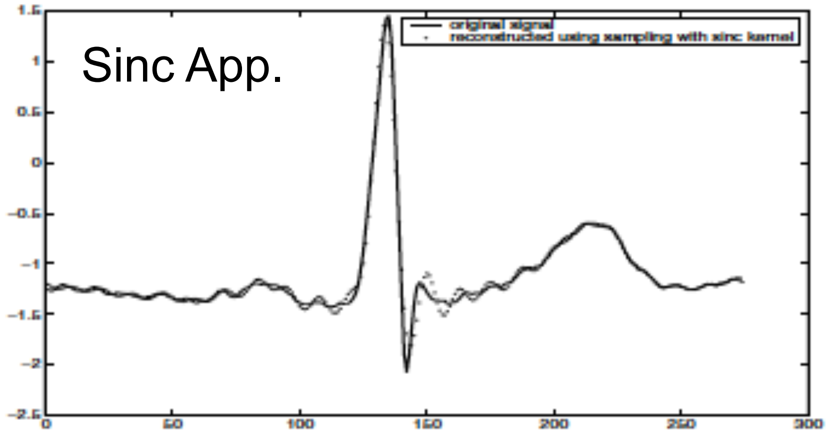
(a)



(a)



(b)



(b)

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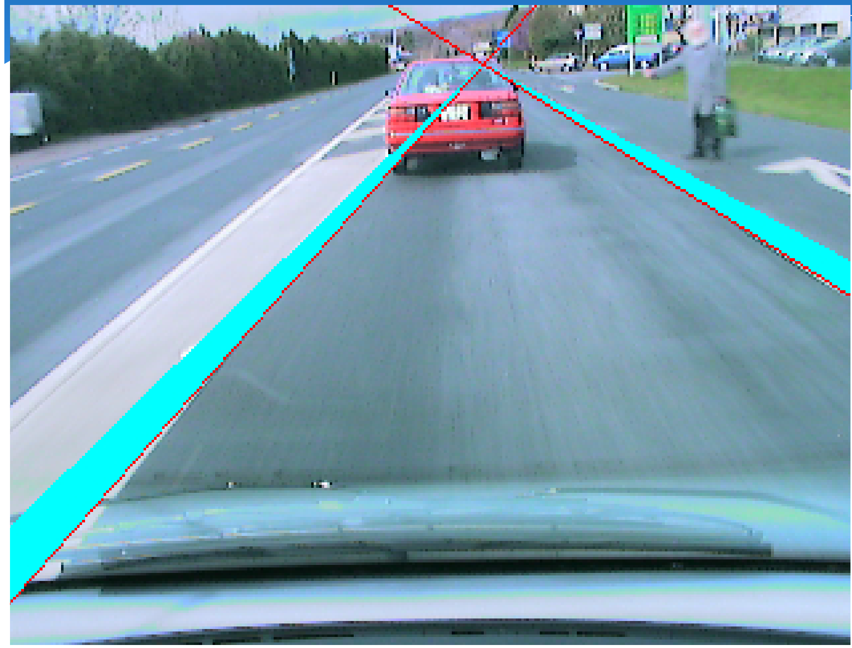
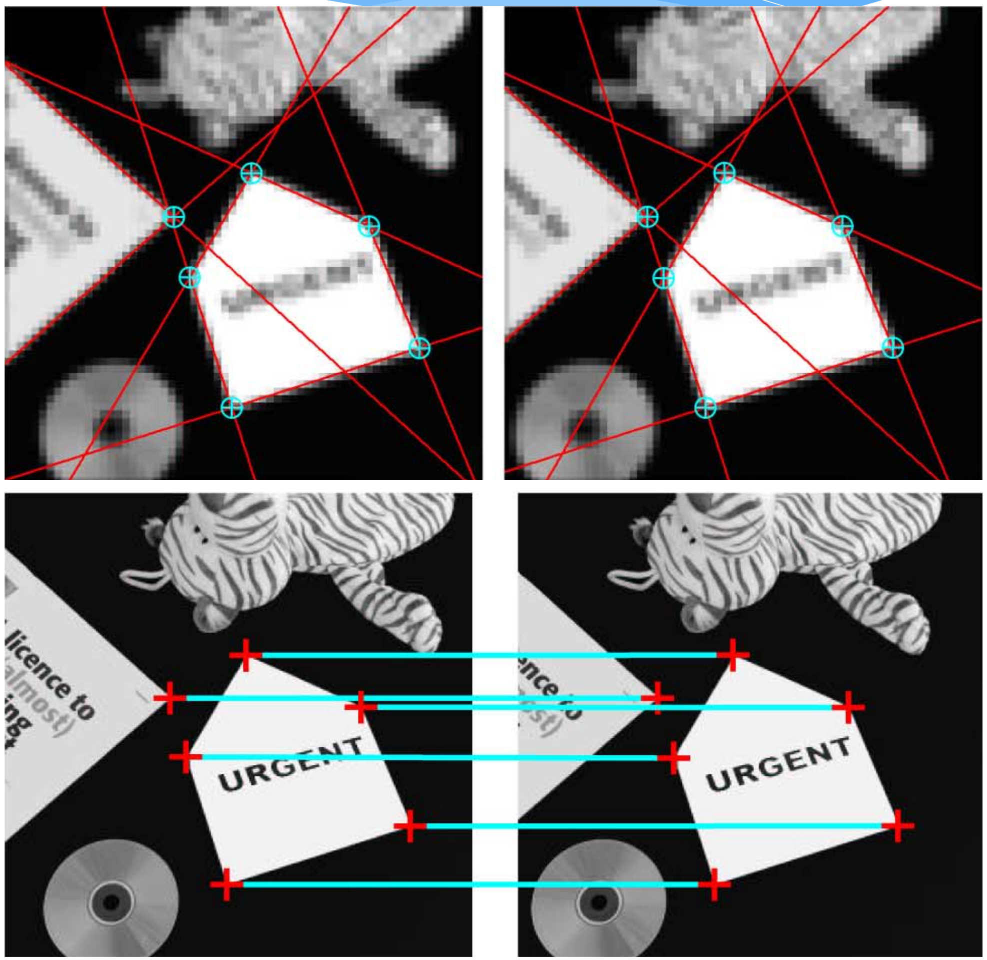
Line-Edge Extraction



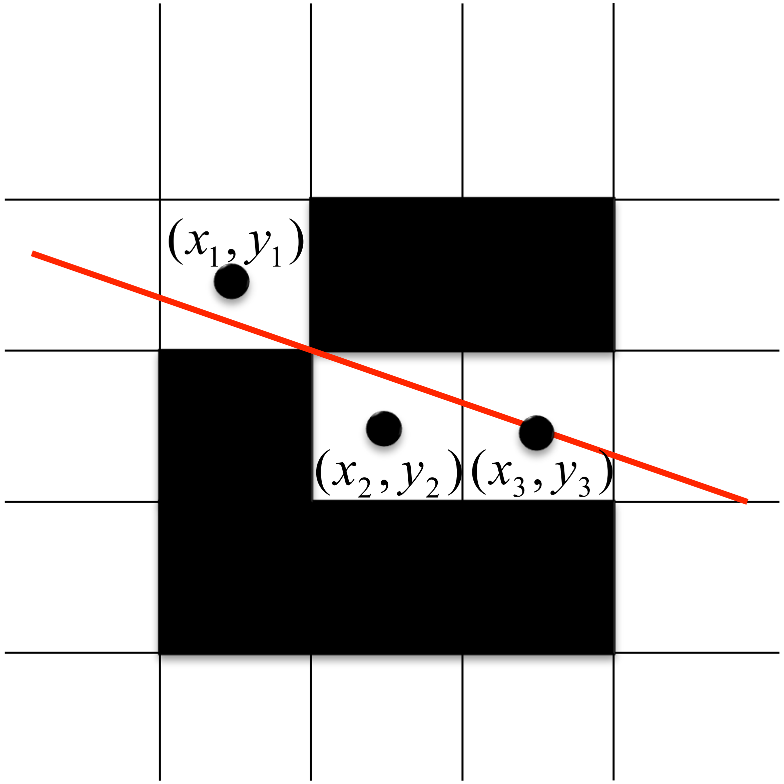
Navigation by lane-detection

Registraction for super-resolution

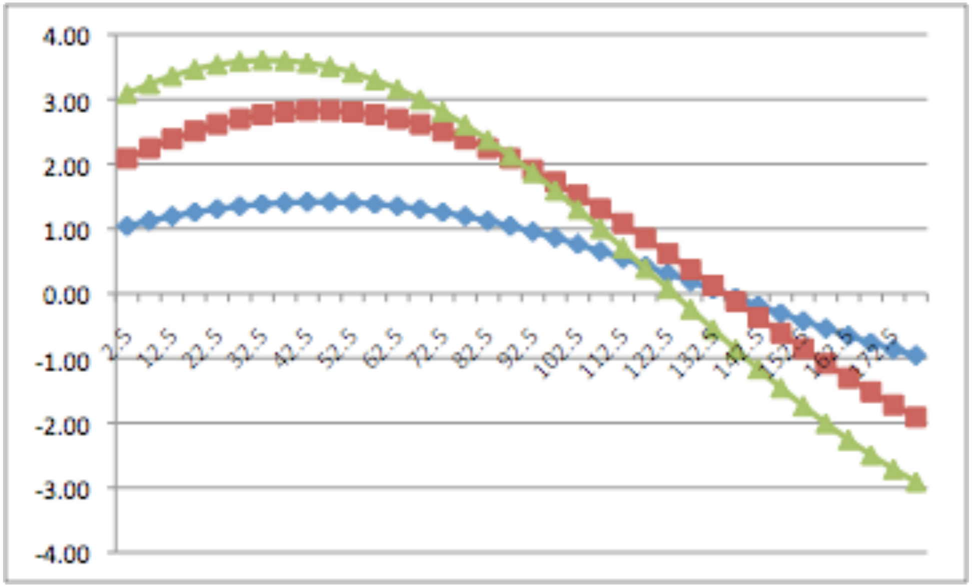


Standard Method: Hough Trans.

$$\rho = x_n \cos\theta + y_n \sin\theta$$



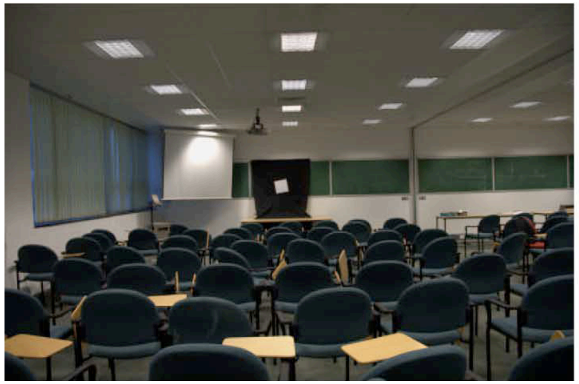
Preciseness degrades as resolution decreases

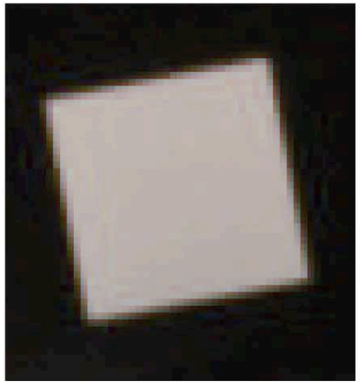


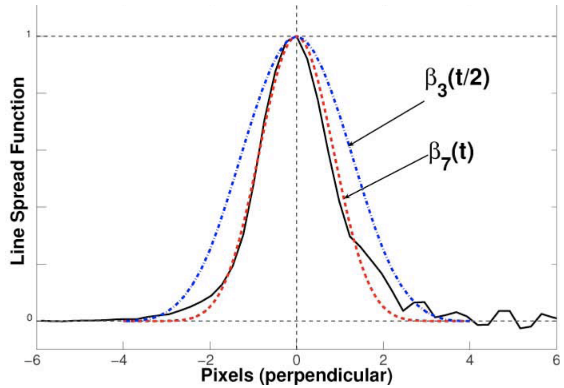
Trade-off between sensitivity and preciseness

New Approach: Sampling Theory

* Pixel acquisition process is modeled by spline functions







(Baboulaz et al., 2009)

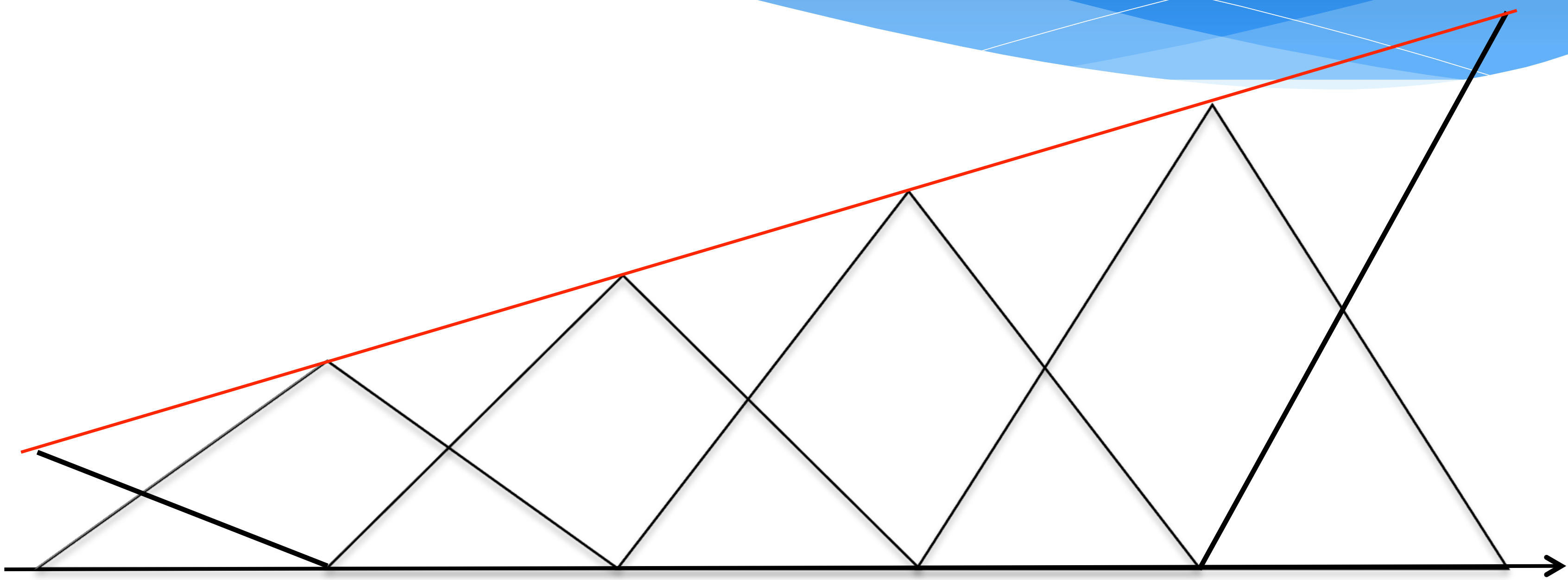
$$h[m,n] = \langle f(x,y), \psi(x-m)\psi(y-n) \rangle$$

Pixel value

Continuous image

Point Spread Function Model
By B-spline or E-spline

Polynomial Reconstruction by B-spline



$$\ldots + c_0^{(p)}\beta_P(t) + c_1^{(p)}\beta_P(t-1) + c_2^{(p)}\beta_P(t-2) + \ldots = t^p$$

Moment of Image

$$g[n] = \left\langle f(t), \beta_P(t-n) \right\rangle$$

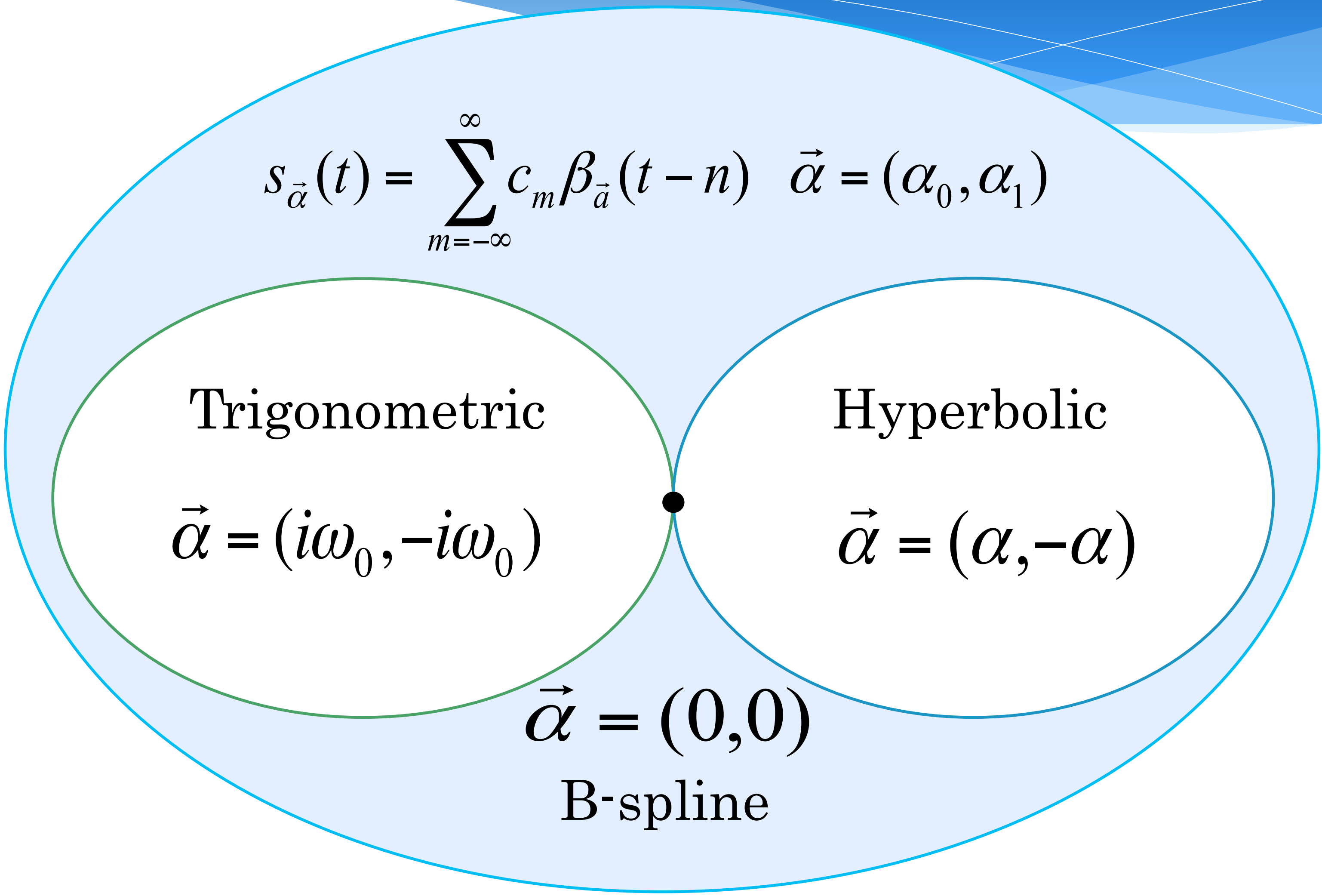
$$\sum_{n=-\infty}^{\infty} c_n^{(p)} g[n] = \sum_{n=-\infty}^{\infty} c_n^{(p)} \left\langle f(t), \beta_P(t-n) \right\rangle$$

$$= \left\langle f(t), \sum_{n=-\infty}^{\infty} c_n^{(p)} \beta_P(t-n) \right\rangle$$

$$= \left\langle f(t), t^p \right\rangle$$

$$= \int_{-\infty}^{\infty} f(t) t^p \, dt$$

E-spline of Degree 1



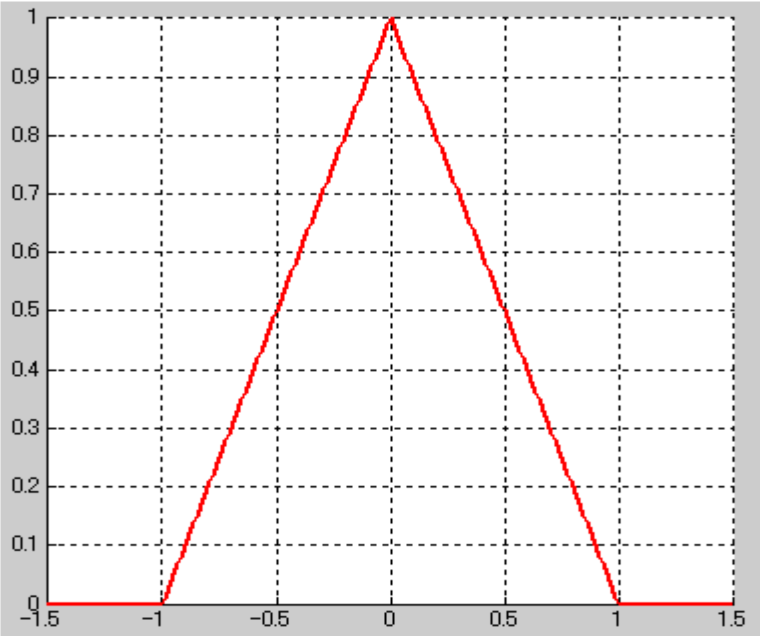
B-Spline & tri. E-Spline

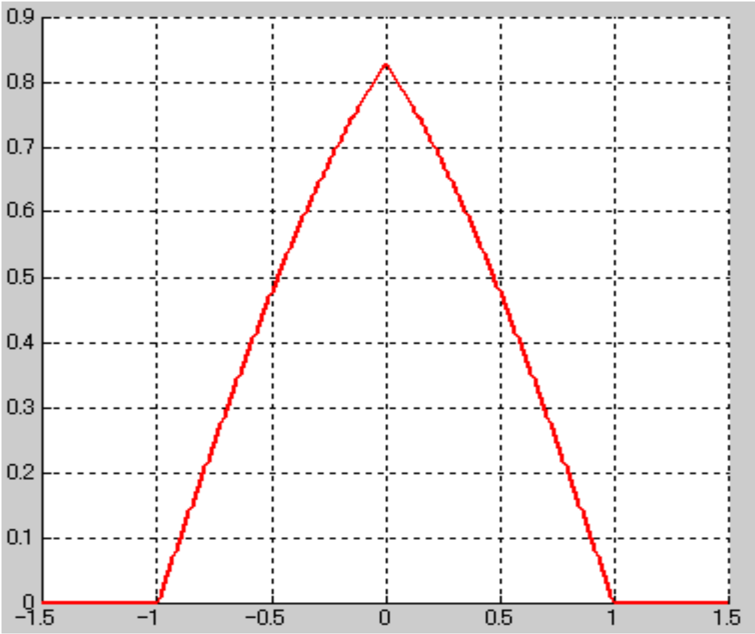
B-Spline of Degree 1

$$\beta_1(t) = \begin{cases} 1+t & (-1<t<0) \\ 1-t & (0<t<1) \\ 0 & (|t|>0) \end{cases}$$

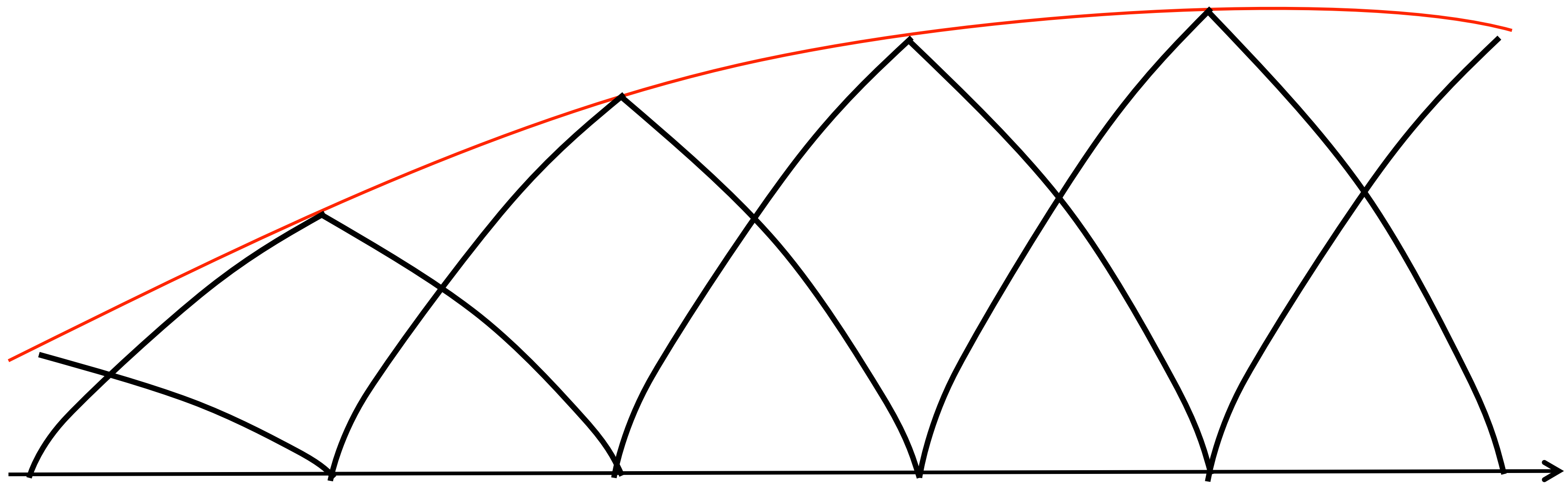
Trigonometric E-Spline

$$\beta_{\vec{\alpha}}(t) = \begin{cases} \sin\omega_0(1+t)/\omega_0 & (-1<t<0) \\ \sin\omega_0(1-t)/\omega_0 & (0<t<1) \\ 0 & (|t|>0) \end{cases}$$





Sinusoid Reconstruction by Trigonometric E-spline



$$\ldots + c_0^{(\alpha_p)}\beta_{\vec{\alpha}}(t) + c_1^{(\alpha_p)}\beta_{\vec{\alpha}}(t-1) + c_2^{(\alpha_p)}\beta_{\vec{\alpha}}(t-2) + \ldots = e^{\alpha_p t}$$

Exponential Moment of Image

$$g[n] = \left\langle f(t), \beta_P(t-n) \right\rangle$$

$$\sum_{n=-\infty}^{\infty} c_n^{(p)} g[n] = \sum_{n=-\infty}^{\infty} c_n^{(p)} \left\langle f(t), \beta_P(t-n) \right\rangle$$

$$= \left\langle f(t), \sum_{n=-\infty}^{\infty} c_n^{(p)} \beta_P(t-n) \right\rangle$$

$$= \left\langle f(t), e^{\alpha_p t} \right\rangle$$

$$= \int_{-\infty}^{\infty} f(t) e^{\alpha_p t}\, dt$$

Procedure for Edge Extarction

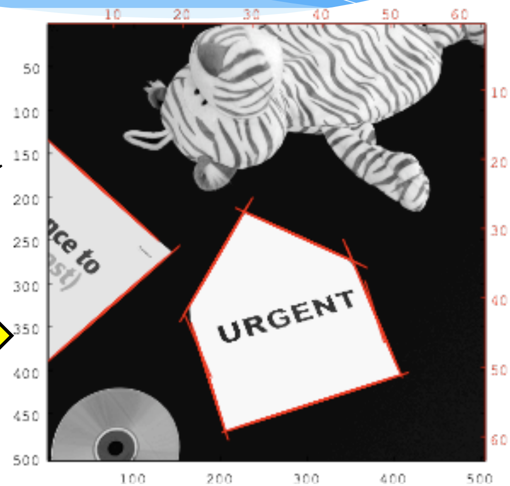
(Baboulaz et al., 2009, Hirabayashi et al., 2010)

Input image

Detection

Detected pixels

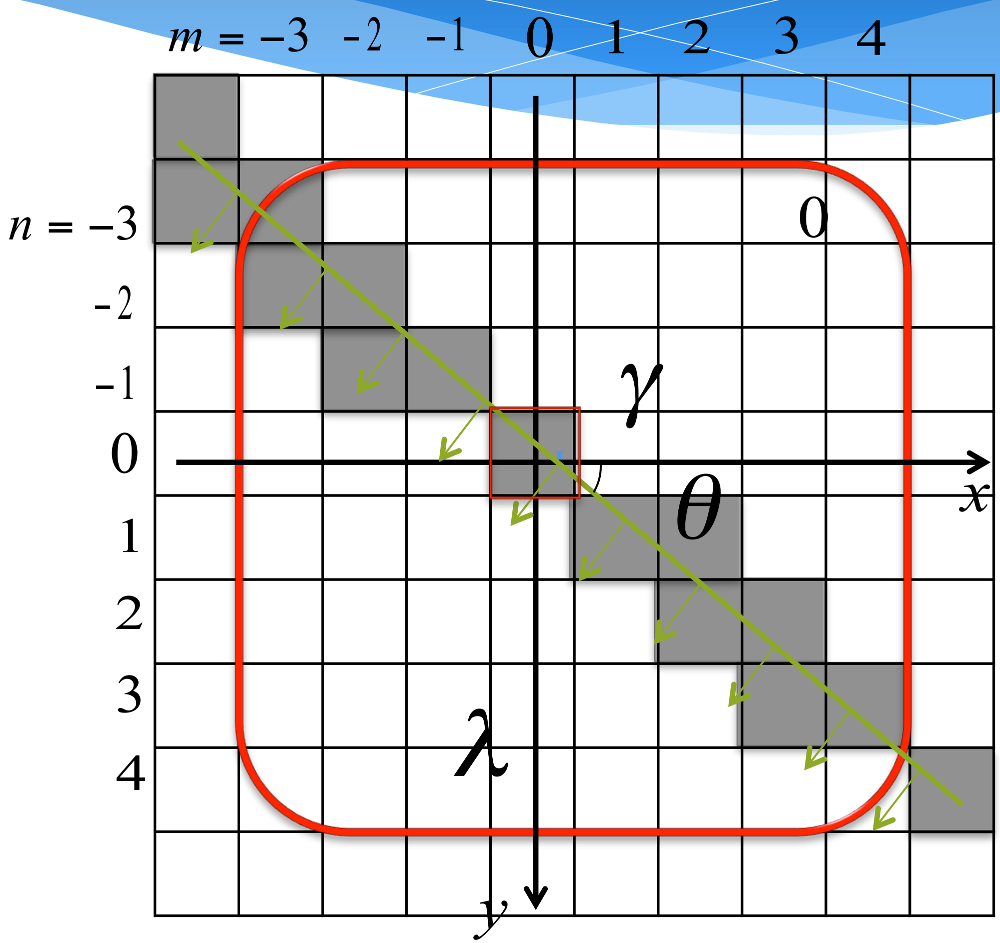
Extraction



Extracted lines

Extraction of Local Area of 8*8

For all detected pixels by Canny detector, we repeat line-edge extraction procedure. To do so, we extract 8*8 local image around the focusing pixel.



Step Line-Edge Representation

$$f(x, y) = \lambda u(-x \sin\theta + y \cos\theta + \gamma \sin\theta)$$

$$u(t) = \begin{cases} 0 & (t < 0) \\ 1 & (t \geq 0) \end{cases}$$

Step Line-Edge Representation

$$f(x,y) = \lambda u(-x\sin\theta + y\cos\theta + \gamma\sin\theta)$$

$$u(t) = \begin{cases} 0 & (t < 0) \\ 1 & (t \geq 0) \end{cases}$$

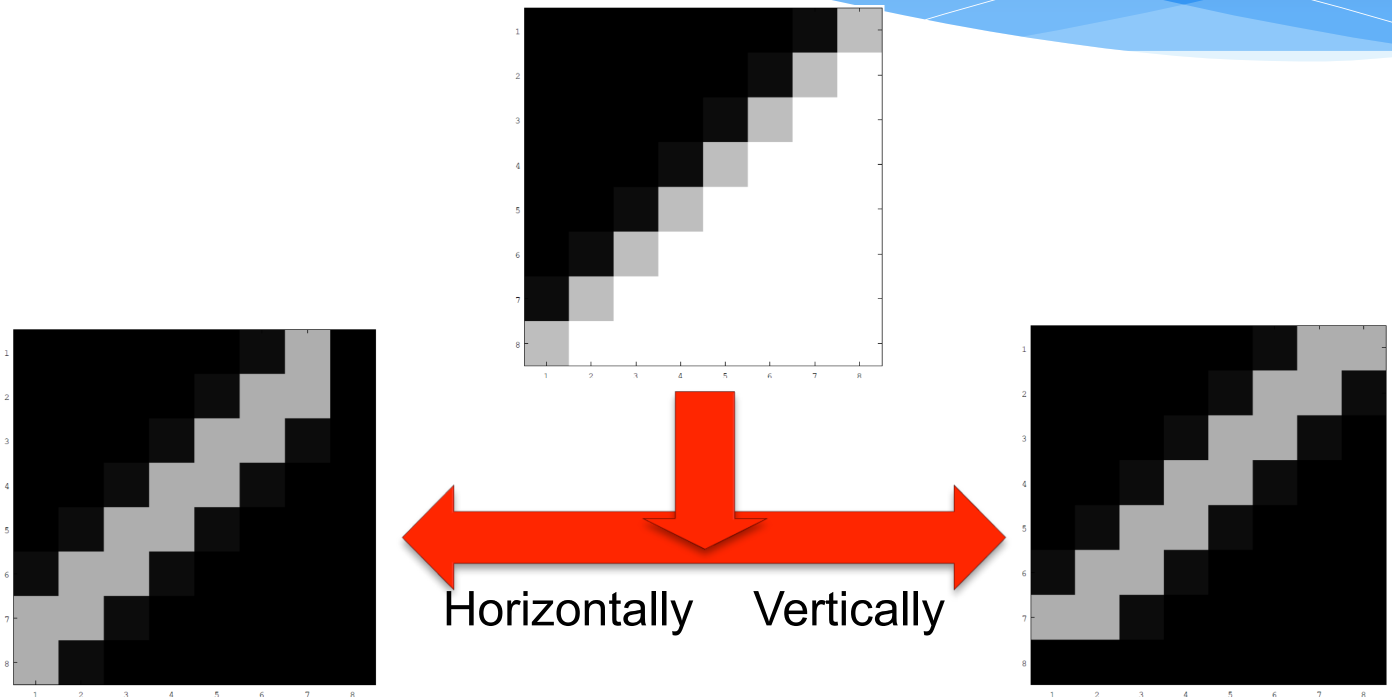
$$g[m,n] = \langle f(x,y), \psi(x-m)\psi(y-n)\rangle + \varepsilon[m,n]$$

$\psi(t)$: Trigonometric E-spline

$\varepsilon[m,n]$: Additive noise

Step 1: Differentiated Samples

$$g[m,n]$$



$$d_H[m,n] = g[m+1,n] - g[m,n]$$

$$d_V[m,n] = g[m,n+1] - g[m,n]$$

Differentiated Sample & Derivative of Continuous Image

$$d_H[m,n] = g[m+1,n] - g[m,n]$$

$$= \left\langle \frac{\partial f(x,y)}{\partial x}, \psi_+(x-m)\psi(y-n) \right\rangle \quad (15)$$

$$\psi_+(t) = (\psi * \beta_{\alpha_2})(t-0.5),$$

$$\beta_{\alpha_2}(t) = \begin{cases} 1 & (|t| < 0.5) \\ 0 & (|t| \geq 0.5) \end{cases}$$

Cf.

$$g[m,n] = \langle f(x,y), \psi(x-m)\psi(y-n) \rangle$$

Proof

Let $\Phi(t) = \int \psi(t)dt$. Then, it follows that

$$\begin{aligned}
\psi_+(t) &= \int_{-\infty}^{\infty} \beta_{\alpha_2}(t')\psi(t-0.5-t')dt' \\
&= \int_{-0.5}^{0.5} \psi(t-t')dt' = \Phi(t) - \Phi(t-1).
\end{aligned}$$

Differentiating both sides of this equation yields

$$\frac{d}{dt}\psi_+(t) = \psi(t) - \psi(t-1).$$

Hence, it follows that

$$d_H[m,n] = \langle f(x,y), \{\psi(x-m-1) - \psi(x-m)\}\psi(y-n)\rangle$$

$$\begin{aligned}
&= \left\langle f(x,y), \{-\frac{d}{dt}\psi_+(x-m)\}\psi(y-n)\right\rangle \\
&= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\left\{\frac{d}{dx}\psi_+(x-m)\right\}\psi(y-n)dxdy \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial f(x,y)}{\partial x}\psi_+(x-m)\psi(y-n)dxdy \\
&= \left\langle \frac{\partial f(x,y)}{\partial x}, \psi_+(x-m)\psi(y-n)\right\rangle,
\end{aligned}$$

which implies (15). Equation (16) can be shown similarly. ■

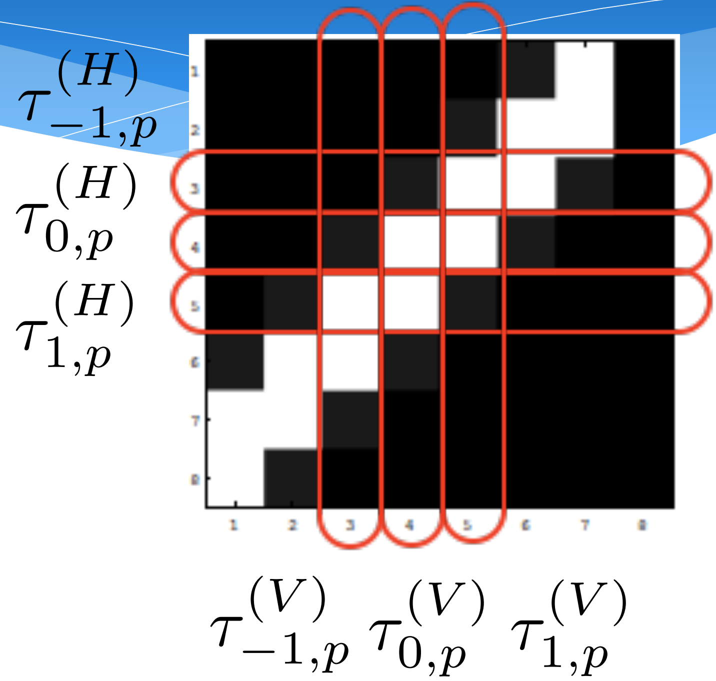
Step 2: Product-Sum

$$d_H[m,n] = g[m+1,n] - g[m,n]$$

$$\tau^{(H)}_{n,p} = \sum_{m=-3}^{3} C^{(\alpha_p)}_m d_H[m,n] \quad (17)$$

$$d_V[m,n] = g[m,n+1] - g[m,n]$$

$$\tau^{(V)}_{m,p} = \sum_{m=-3}^{3} C^{(\alpha_p)}_n d_V[m,n]$$



$$p = 0, 1, 2$$

Closed Form for Product-Sum

$$\tau_{n,p}^{(H)} = \lambda\mu_{n,p}^{(H)}(\theta,\gamma)$$

$$\mu_{n,p}^{(H)}(\theta,\gamma) = -\mathrm{sgn}(\sin\theta)e^{\alpha_p(\gamma+\frac{n}{\tan\theta}-\frac{1}{2})}\Psi(\frac{\alpha_p}{\tan\theta}) \quad (21)$$

$$\tau_{m,p}^{(V)} = \lambda\mu_{m,p}^{(V)}(\theta,\gamma)$$

$$\mu_{m,p}^{(V)}(\theta,\gamma) = \mathrm{sgn}(\cos\theta)e^{\alpha_p\{-(\gamma-m)\tan\theta-\frac{1}{2}\}}\Psi(\alpha_p\tan\theta)$$

$$\Psi(s) = \int_{-\infty}^{\infty}\psi(t)e^{st}dt \qquad \mathrm{sgn}(t) = \begin{cases} 1 & (t>0), \\ 0 & (t=0), \\ -1 & (t<0) \end{cases}$$

Proof

Proof: Because of (12), we have

$$\frac{\partial f(x,y)}{\partial x} = -\lambda\delta(-x\sin\theta + y\cos\theta + \gamma\sin\theta)\sin\theta,$$

where $\delta(t)$ is Dirac's delta function. Substituting this relation and (15) into (17) yields

$$\tau_{n,p}^{(H)} = \begin{cases} -\lambda e^{\alpha_p(\gamma+n/\tan\theta-\frac{1}{2})}\Psi(\alpha_p/\tan\theta) & (\sin\theta > 0), \\ 0 & (\sin\theta = 0), \\ \lambda e^{\alpha_p(\gamma+n/\tan\theta-\frac{1}{2})}\Psi(\alpha_p/\tan\theta) & (\sin\theta < 0), \end{cases}$$

which are combined into (21).

Step 2: Product-Sum (cnt'd)

Horizontally

$$
\begin{array}{c c}
 & p = 0 \quad\quad 1 \quad\quad 2 \\
\begin{array}{r} n = -1 \\ 0 \\ 1 \end{array} &
\begin{pmatrix}
\tau^{(H)}_{-1,0} & \tau^{(H)}_{-1,1} & \tau^{(H)}_{-1,2} \\
\tau^{(H)}_{0,0} & \tau^{(H)}_{0,1} & \tau^{(H)}_{0,2} \\
\tau^{(H)}_{1,0} & \tau^{(H)}_{1,1} & \tau^{(H)}_{1,2}
\end{pmatrix}
\end{array}
$$

Vertically

$$
\begin{array}{c c}
\begin{array}{r} m = -1 \\ 0 \\ 1 \end{array} &
\begin{pmatrix}
\tau^{(V)}_{-1,0} & \tau^{(V)}_{-1,1} & \tau^{(V)}_{-1,2} \\
\tau^{(V)}_{0,0} & \tau^{(V)}_{0,1} & \tau^{(V)}_{0,2} \\
\tau^{(V)}_{1,0} & \tau^{(V)}_{1,1} & \tau^{(V)}_{1,2}
\end{pmatrix}
\end{array}
$$

Criterion for Param. Estimation

$$J_o(\theta,\gamma,\lambda) = \|\lambda\boldsymbol{\mu}(\theta,\gamma) - \boldsymbol{\tau}\|^2$$

$$\boldsymbol{\mu}(\theta,\gamma) = \begin{pmatrix} \mu^{(H)}_{-1,0}(\theta,\gamma) \\ \mu^{(H)}_{-1,1}(\theta,\gamma) \\ \mu^{(H)}_{-1,2}(\theta,\gamma) \\ \mu^{(H)}_{0,0}(\theta,\gamma) \\ \vdots \\ \mu^{(V)}_{0,2}(\theta,\gamma) \\ \mu^{(V)}_{1,0}(\theta,\gamma) \\ \mu^{(V)}_{1,1}(\theta,\gamma) \\ \mu^{(V)}_{1,2}(\theta,\gamma) \end{pmatrix} \qquad \boldsymbol{\tau} = \begin{pmatrix} \tau^{(H)}_{-1,0} \\ \tau^{(H)}_{-1,1} \\ \tau^{(H)}_{-1,2} \\ \tau^{(H)}_{0,0} \\ \vdots \\ \tau^{(V)}_{0,2} \\ \tau^{(V)}_{1,0} \\ \tau^{(V)}_{1,1} \\ \tau^{(V)}_{1,2} \end{pmatrix}$$

Optimization in Terms of λ

For fixed θ, γ

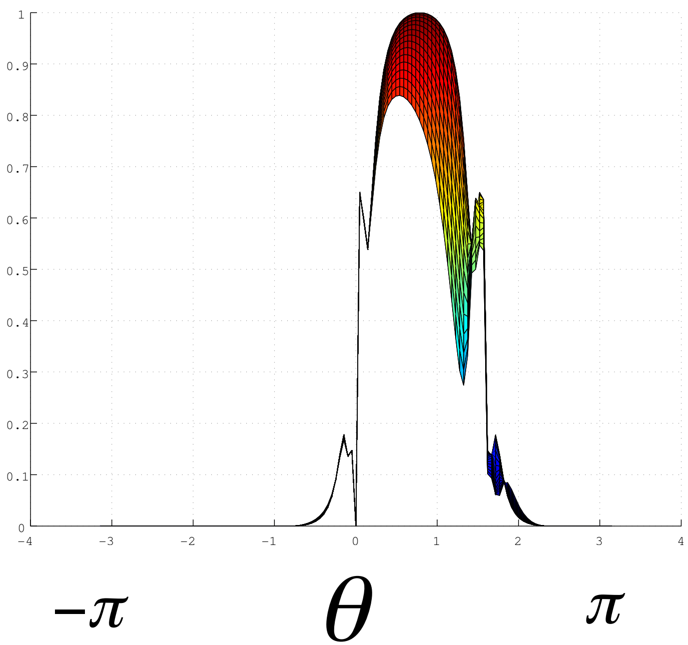
$$\begin{aligned} J_o(\theta,\gamma,\lambda) &= \|\boldsymbol{\mu}(\theta,\gamma)\|^2 \left(\lambda - \frac{\langle \boldsymbol{\tau}, \boldsymbol{\mu}(\theta,\gamma)\rangle}{\|\boldsymbol{\mu}(\theta,\gamma)\|^2}\right)^2 \\ &\quad - \frac{\{\langle \boldsymbol{\tau}, \boldsymbol{\mu}(\theta,\gamma)\rangle\}^2}{\|\boldsymbol{\mu}(\theta,\gamma)\|^2} + \|\boldsymbol{\tau}\|^2 \\ &\geq \|\boldsymbol{\tau}\|^2 \left[1 - \frac{\{\langle \boldsymbol{\tau}, \boldsymbol{\mu}(\theta,\gamma)\rangle\}^2}{\|\boldsymbol{\tau}\|^2 \|\boldsymbol{\mu}(\theta,\gamma)\|^2}\right]. \end{aligned}$$

$$J(\theta,\gamma) = \frac{\{\langle \boldsymbol{\tau}, \boldsymbol{\mu}(\theta,\gamma)\rangle\}^2}{\|\boldsymbol{\tau}\|^2 \|\boldsymbol{\mu}(\theta,\gamma)\|^2}$$

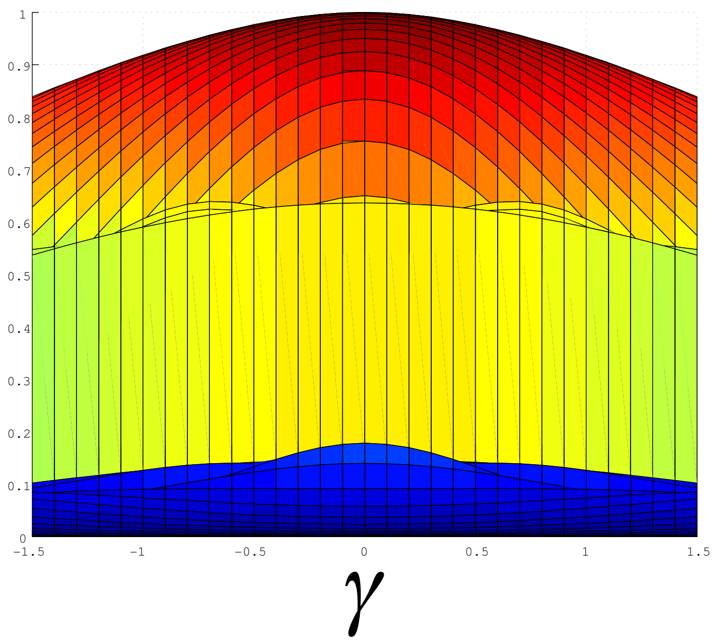
Value of $J(\vartheta, \gamma)$

In case of $\theta = \dfrac{\pi}{4}, \gamma = 0$

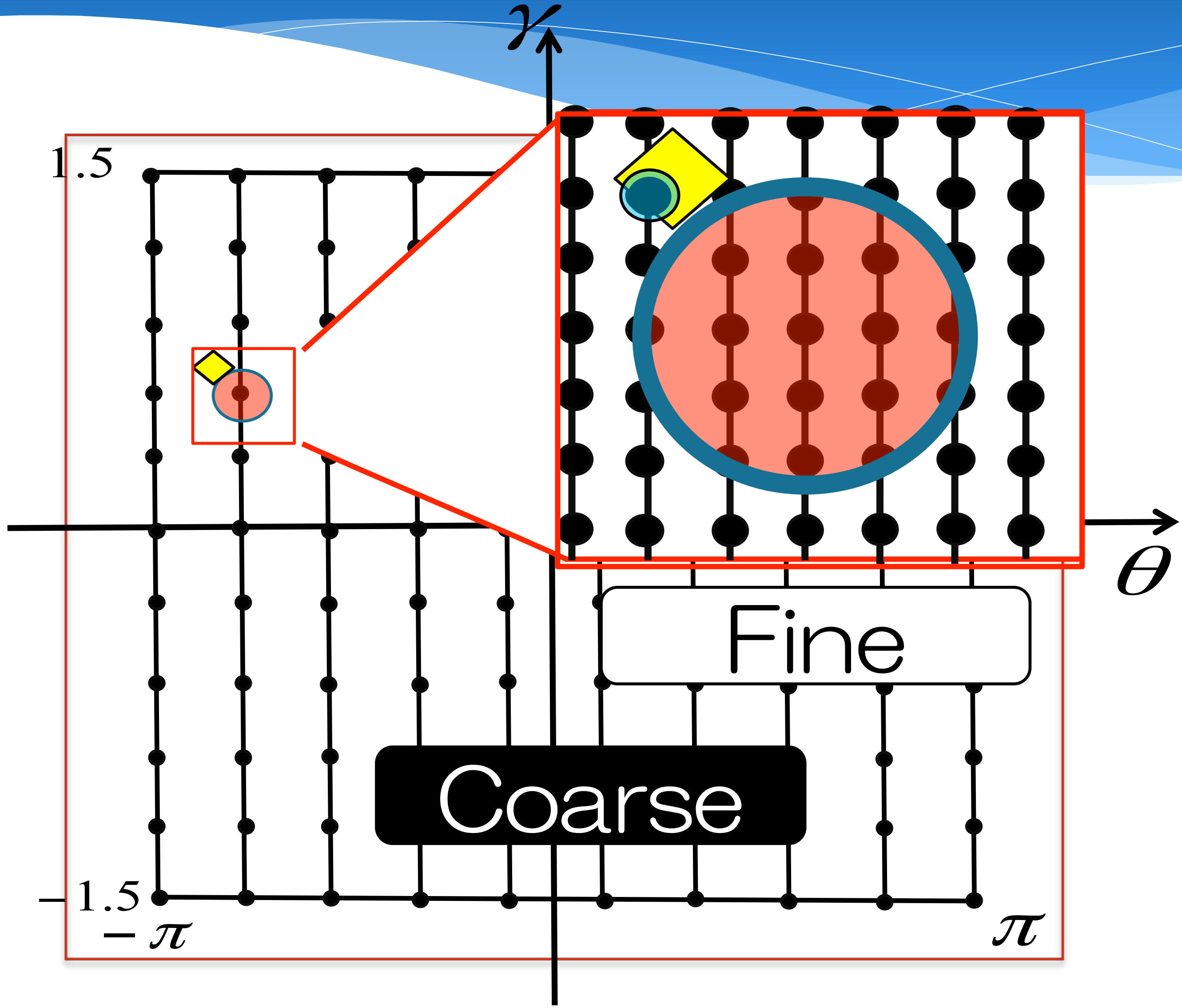
View along θ axis



View along γ axis

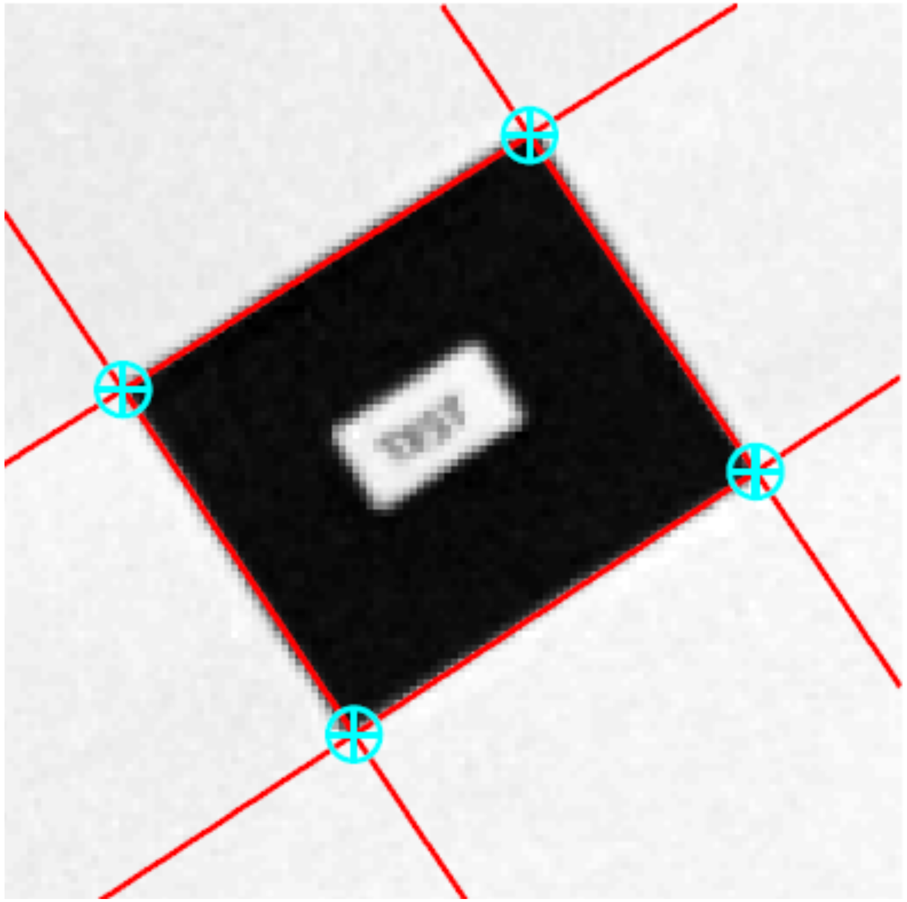


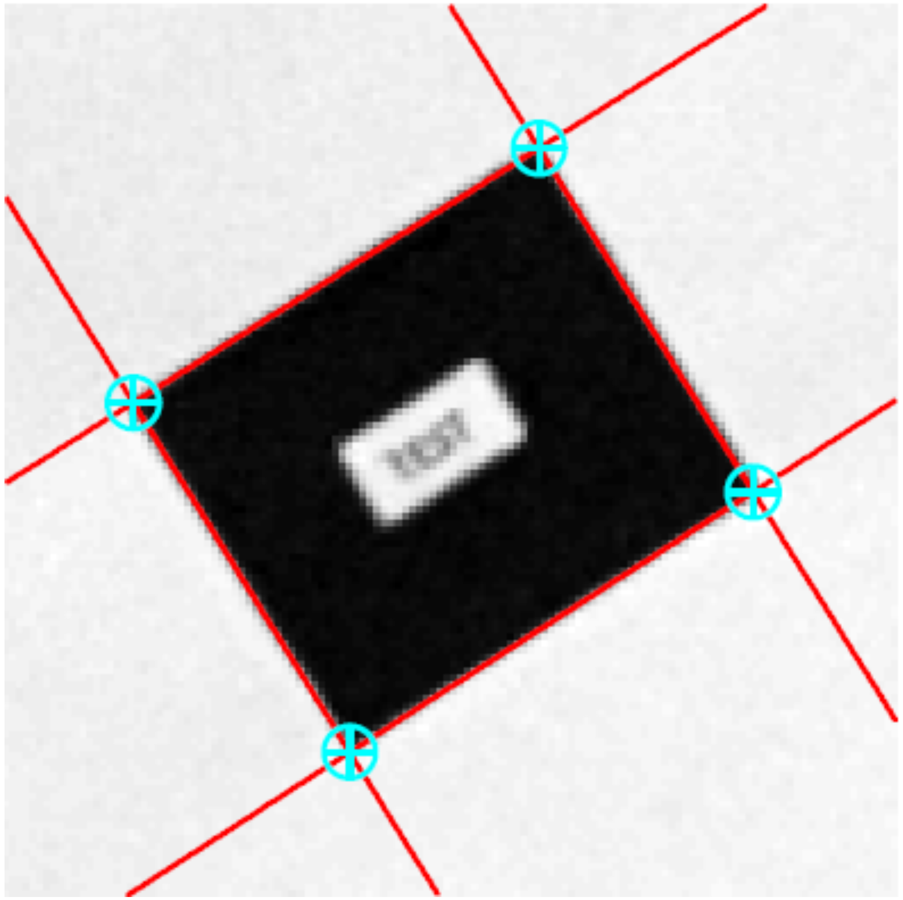
Coarse to Fine Search



Simulation Results

Panasonic Lumix DMC-GF2 (focal length14mm, F2, 1/60s, ISO200)

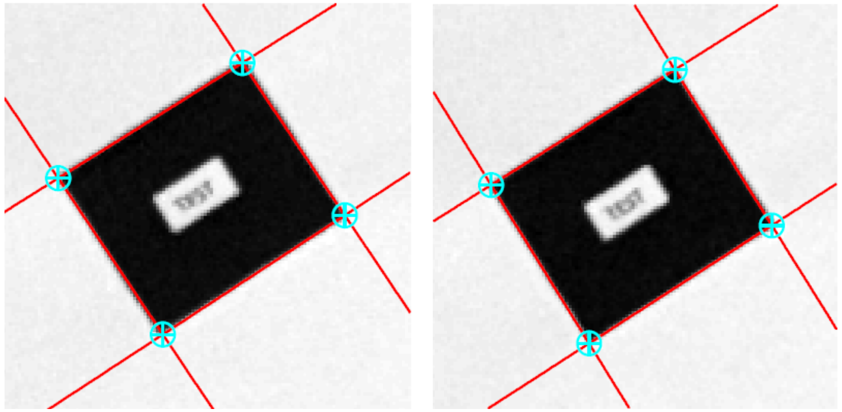


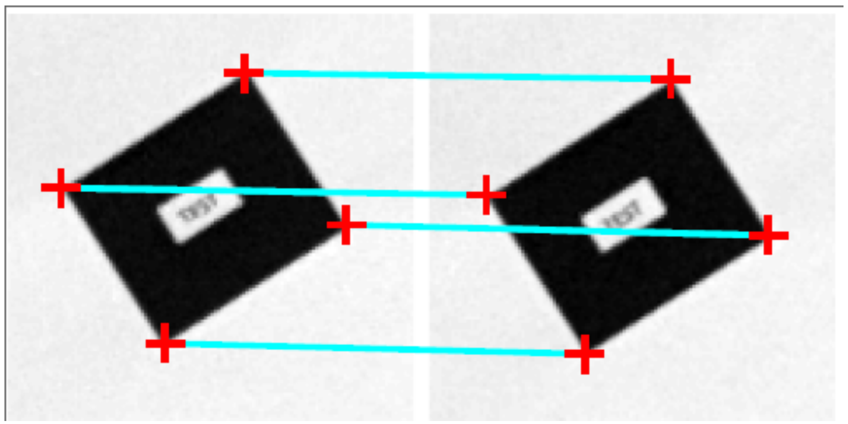


Super-Resolution

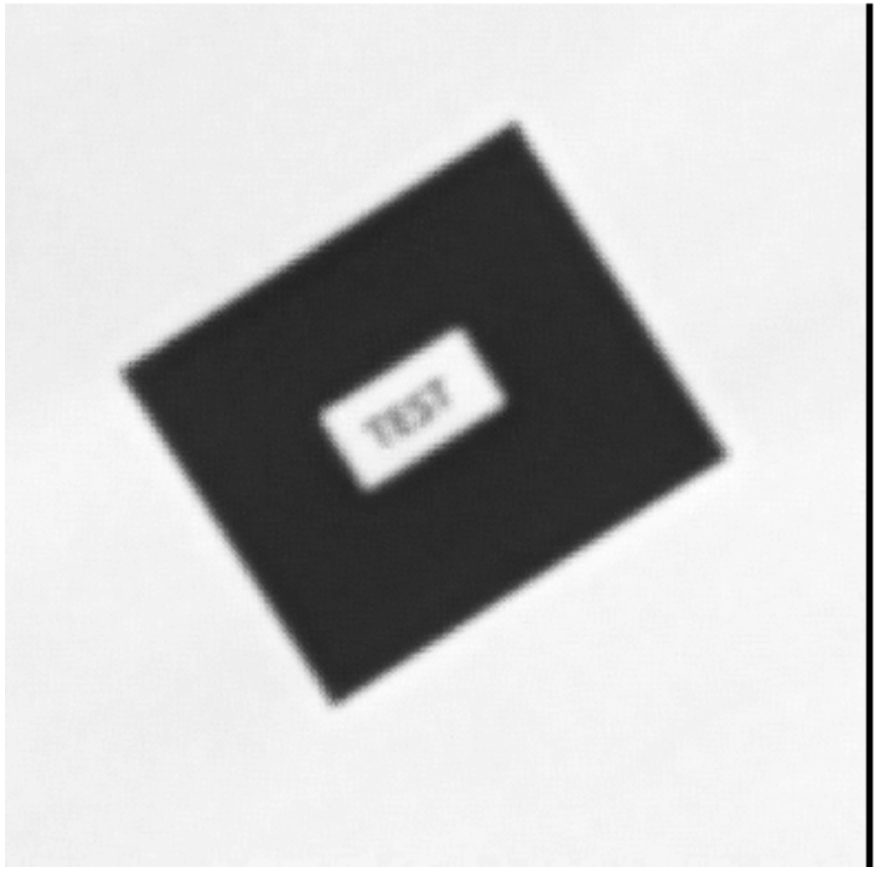
Panasonic Lumix DMC-GF2 (focal length 14mm, F2, 1/60s, ISO200)

Line-edge extraction results



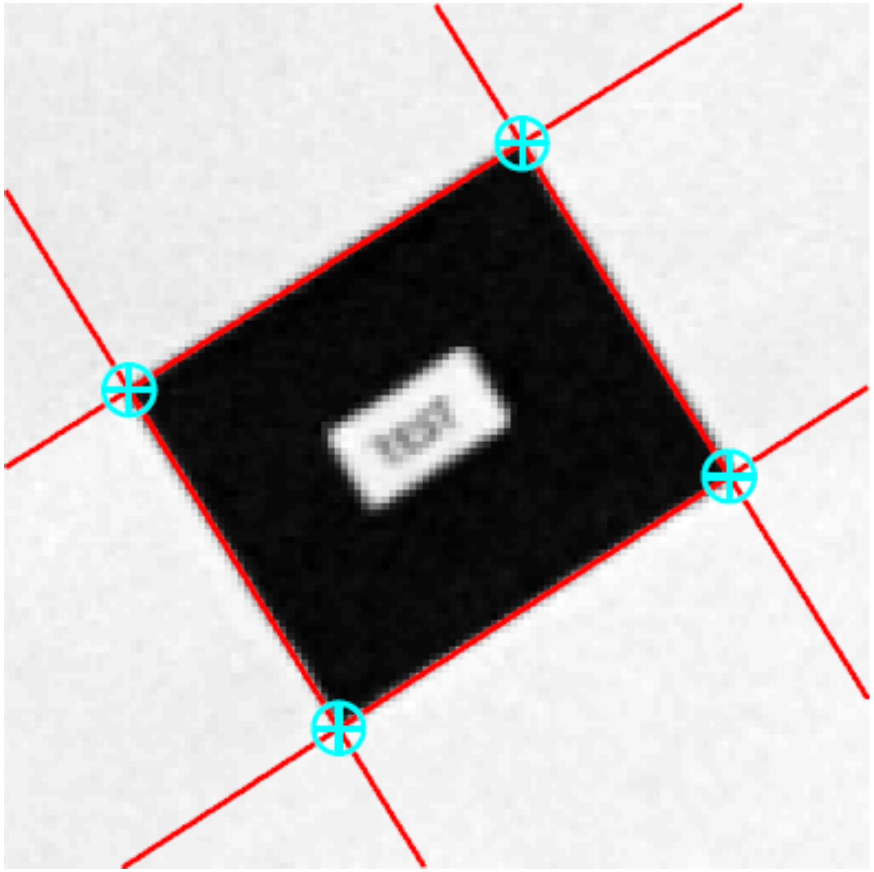


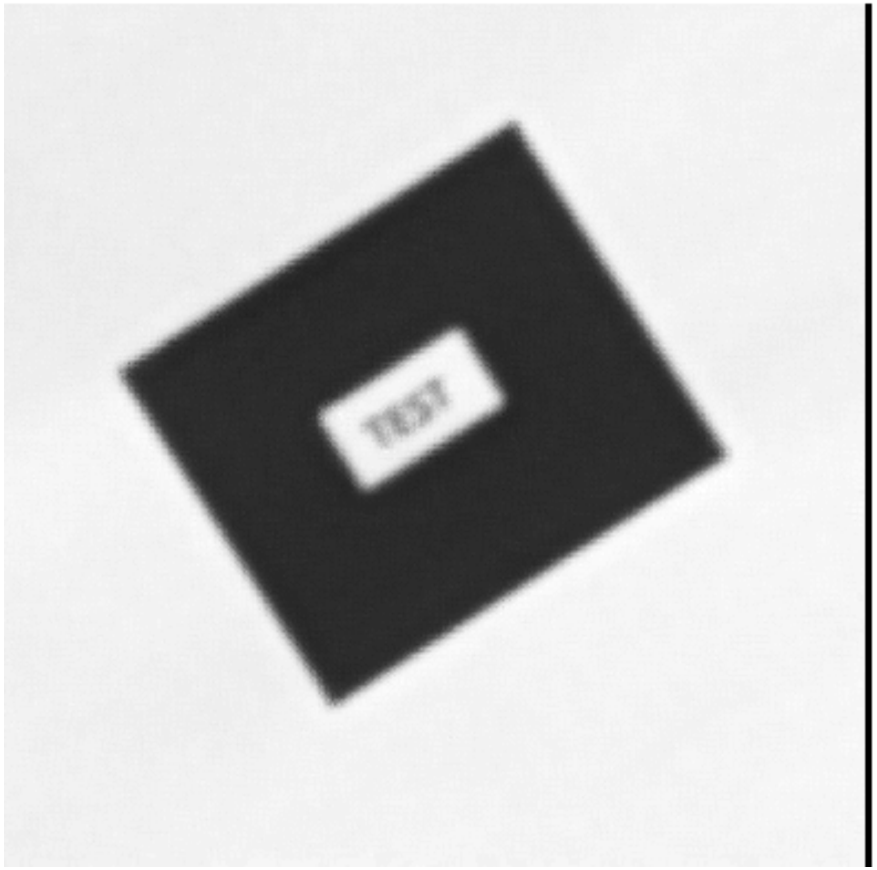
Matching by RANSAC algorithm



Super-resolved from 16 images

Super-Resolution



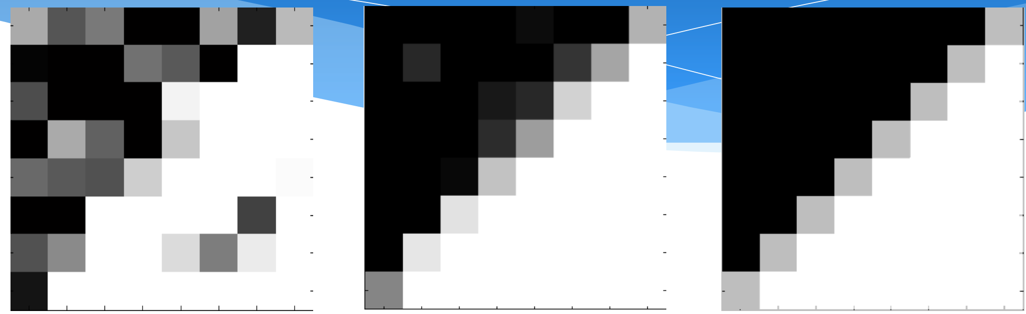


Super-resolved from 16 images

Resilience to Noise

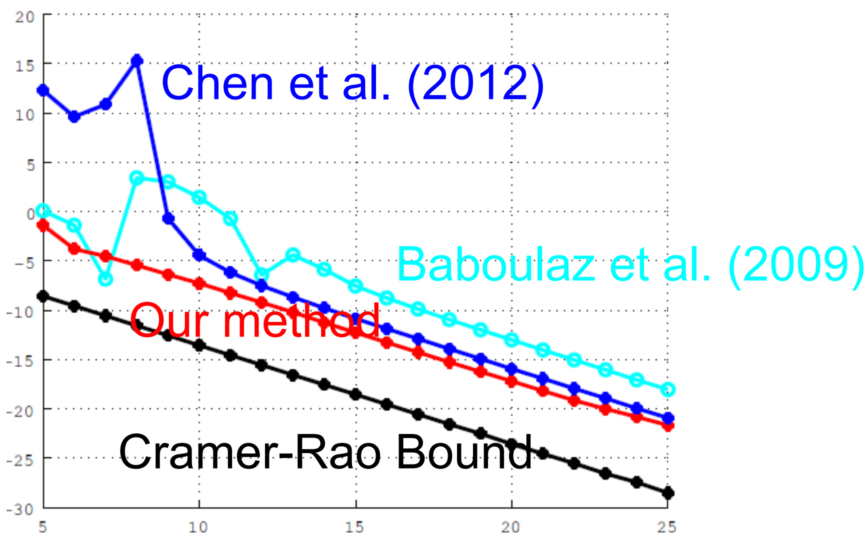
Signal/noise ratio

$$SNR = 10\log(\frac{\lambda}{\sigma})$$

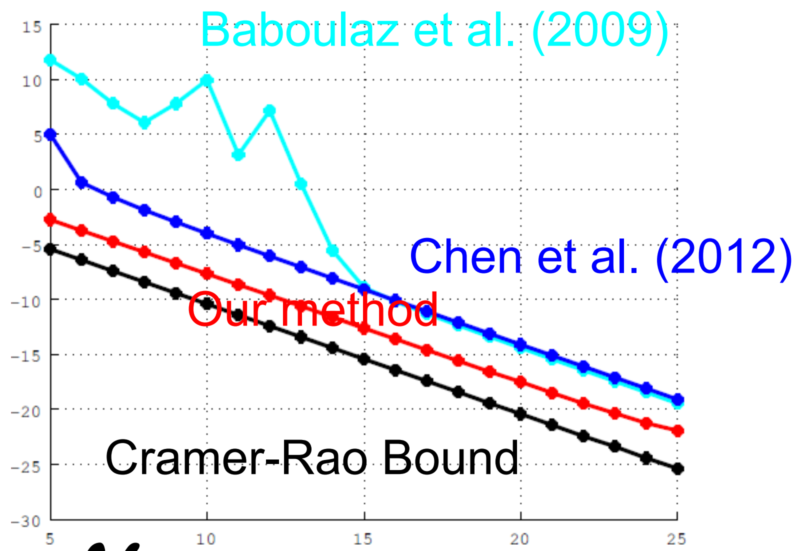


強 5dB 10dB 25dB 弱

MSE[dB]



$\tan\theta$



γ

Summary

- Sampling signals with finite rate of innovation
  - Rate of innovation is defined by the number of unknown parameters.
  - The main signal is the stream of Diracs.
  - Signal is sampled through the filter
  - Reconstruction is done by annihilating filter for noiseless case while optimization technique is used for noisy case.
- Applications
  - Compressive sampling for ECG signals
  - Step line-edge extraction

Reference

* M. Vetterli, P. Marziliano, and T. Blu, “Sampling signals with finite rate of innovation,” IEEE Trans. Signal Processing , vol. 50, no. 6, pp. 1417–1428, June 2002.
* T. Blu, P.L. Dragotti, M. Vetterli, P. Marziliano, and L. Coulot, “Sparse sampling of signal innovations,” IEEE Signal Processing Magazine , vol. 25, no. 2, pp. 31–40, March 2008.
* Akira Hirabayashi, “Sampling and reconstruction of periodic piecewise polynomials using sinc kernel,” IEICE Trans. on Fundamentals of Electronics, Communications and Computer Sciences, vol. E95-A, no. 1, pp.322-329, Jan. 2012.