6th IM-Workshop on

Applied Approximation, Signals, and Images

Bernried (Germany)
February 20–24, 2023

PROGRAM & ABSTRACTS

Organizers:
Costanza Conti
Mariantonia Cotronei
Nira Dyn
Brigitte Forster
Tomas Sauer
Emil Žagar

PROGRAM

Monday, February 20	
09:00– 09:10	*Welcome & Opening*
09:10–10:30	Getting to know each other: Mathematical speed dating I
10:30–11:00	**Coffee break**
11:00–12:30	Getting to know each other: Mathematical speed dating II
15:00–15:30	**Coffee & cake**
15:30–16:30	Ognyan Kounchev: *Application of spline based versions of SEIR model to Covid-19 spread*
16:30–17:00	Francesco Marchetti: *Stable mapped polynomial interpolation through* (β, γ)*-Chebyshev points*

Tuesday, February 21	
08:30–09:30	Ulrich Reif: *Prospects of Subdivision in Isogeometric Analysis*
09:30–10:30	Alexander Dietz: *Integration on Subdivision Surfaces*
10:30–11:00	**Coffee break**
11:00–12:00	Nira Dyn: *Multivariate* C^∞ *compactly supported basic limit functions*
12:00–12:30	Mariantonia Cotronei: *Reconstruction of binary images through Bernstein polynomials and subdivision schemes*
15:00–15:30	**Coffee & cake**
15:30–16:30	Annie Cuyt: *Exponential Analysis: Solving open problems and Unlocking new potential*
16:30–17:00	Nuha Diab: *Decimated Prony's Method for Stable Super-resolution*
17:00–17:30	Rosanna Campagna: *Hyperbolic-polynomial P-splines: who, what, why*

Wednesday, February 22	
09:00–10:00	Shai Dekel: *Phase Retrieval Using Deep Auto-Decoders*
10:00–11:00	Frank Filbir: *Phase Retrieval from Spectrogram Measurements*
11:00–11:30	**Coffee break**
11:30–12:30	Patricia Römer: *Designing an algorithm for low-dose Poisson phase retrieval*
12:30–13:00	Thomas Hangelbroek: *Jackson and Bernstein inequalities for RBF interpolation in high order Sobolev spaces*
	Excursion

Thursday, February 23	
09:00–10:00	Michele Piana: *Numerical approximation for image reconstruction in the 'Spectrometer/Telescope for Imaging X-rays (STIX)' mission on-board 'Solar Orbiter'*
10:00–10:30	Jan Grošelj: *Super-smoothness of* C^1 *cubic Powell–Sabin splines*
10:30–11:00	**Coffee break**
11:00–11:30	Ada Šadl Praprotnik: *Exact sphere representations over Platonic solids based on rational multisided Bézier patches*
11:30–12:00	Hendrik Speleers: *Maximally smooth splines on refined triangulations: Local simplex spline bases*
12:00–12:30	Aleš Vavpetič: *Optimal approximation of spherical squares by tensor product quadratic Bézier patches*
15:00–15:30	**Coffee & cake**
15:30–16:00	Gerlind Plonka: *Spline Representation and Redundancies of One-Dimensional ReLU Neural Network Models*
16:00–16:30	Kathrin Schiermeier: *Learning Filters and Wavelets*

Friday, February 24	
09:00–09:30	Emil Žagar: *Interpolation of planar* G^1 *data by Pythagorean-hodograph cubic biarcs with prescribed arc lengths*
09:30–10:00	Florian Heinrich: *Compensation of part deviations in additive manufacturing*
10:00–10:30	A. Michael Stock: *Sparse Volume Reconstruction Based on Haar Wavelet Techniques*
10:30–11:00	**Coffee break**
11:00–12:00	Tomas Sauer: *The Definite Moment for Multivariate Continued Fractions*
12:00–12:05	*Closing remarks*

Hyperbolic-polynomial P-splines: who, what, why

Rosanna Campagna (University of Campania "Luigi Vanvitelli"), Costanza Conti, Salvatore Cuomo*

Hyperbolic-polynomial P-splines, HP-splines for shortness, are particularly interesting in applications requiring data analysis and forecasting with exponential trends. They generalize the better-known polynomial P-splines, introduced by Eilers and Marx in 2010 [1], a regression model used mainly in applications from statistics. The talk discusses the existence, uniqueness, and reproduction properties of HP-splines [2,3] and provides several examples supporting their practical usage in data analysis. Moreover, we propose a *data-driven* strategy to select the frequency, or exponential, parameter that defines the B-spline basis, specific for HP-splines. The proposed algorithm involves a linear algebra approach for Tikhonov regularization problems adapted to the HP-splines definition [4]. As shown in the numerical experiments, our strategy provides an efficient criterion yielding to HP-splines that better capture the trend suggested by the fitted data.

References

[1] P.H.C. Eilers, B.D. Marx, *Flexible smoothing with B-splines and penalties*, Statistical science 11 (2) (1996) 89-121.

[2] R. Campagna, C. Conti, *Penalized hyperbolic-polynomial splines*, Applied Mathematics Letters, Volume 118, (2021), 107-159.

[3] R. Campagna, C. Conti, *Reproduction capabilities of penalized hyperbolic-polynomial splines*, Applied Mathematics Letters, Volume 132, (2022), 108-133.

[4] R. Campagna, C. Conti, S. Cuomo, *A linear algebra approach to HP-splines frequency parameter selection*,(under review)

`rosanna.campagna@unicampania.it`

Reconstruction of binary images through Bernstein polynomials and subdivision schemes

Mariantonia Cotronei (University Mediterranea of Reggio Calabria), Costanza Conti, Demetrio Labate*

The aim of this talk is to illustrate the research carried out in [1,2] in the direction of realizing an efficient procedure for the stable reconstruction of a class of binary images from a small number of measurements. The image boundary is modelled as the zero locus of a bivariate polynomial and the reconstruction can be set up in terms of linear equations associated to a set of image moments. However, the sensitivity of the moments to noise makes the numerical solution highly unstable. We propose a different formulation of the image moment equations, that can obtained by expressing the algebraic boundary in terms of non separable Bernstein polynomials defined over triangular domains. We also illustrate a strategy for the computation of the coefficients involved in such a formulation by means of refinable function kernels. Our approach is robust to noise, computationally fast and simple to implement, as illustrated by some examples.

References

[1] C. Conti, M. Cotronei, D. Labate, W. Molina, Stable recovery of planar regions with algebraic boundaries in Bernstein form. Adv Comput Math 47, 18 (2021)

[2] C. Conti, M. Cotronei, Triangular Bernstein moment-based identification of algebraic curves, submitted

`mariantonia.cotronei@unirc.it`

EXPONENTIAL ANALYSIS: Solving open problems and Unlocking new potential

Annie Cuyt (University of Antwerp & Shenzen University),*
Wen-shin Lee

The Nyquist constraint [11], which is the digital signal processing equivalent of stating that the argument of a complex exponential $\exp(\phi\Delta)$ with $\phi \in \mathbb{C}$ and $\Delta \in \mathbb{R}^+$ can only be retrieved uniquely under the condition that $|\Im(\phi)|\,\Delta < \pi$, governs signal processing since the beginning of the 20-th century. In the past two decades this constraint was first broken with the use of randomly collected signal samples [8, 2] and later for use with uniform samples [3, 4, 6].

The latter method closely relates to the original exponential fitting algorithm published in 1795 by the French mathematician de Prony [7]. Besides avoiding the Nyquist con straint, the new result also solves a number of remaining open problems in exponential analysis, which we plan to discuss.

In the identification, from given values $f_k \in \mathbb{C}$, of the nonlinear parameters $\phi_1, \dots, \phi_n \in \mathbb{C}$, the linear coefficients $\alpha_1, \dots, \alpha_n \in \mathbb{C}$ and the sparsity $n \in \mathbb{N}$ in the inverse problem

$$\sum_{j=1}^{n}\sum_{j=1}^{n} \alpha_j \exp(\phi_j k\Delta) = f_k, \qquad k = 0, \dots, 2n-1, \dots \quad f_k \in \mathbb{C},\, \Delta \in \mathbb{R}^+ \qquad (1)$$

several cases are considered to be hard [6, 1]:

- When some of the ϕ_j cluster, the identification and separation of these clustered ϕ_j becomes numerically ill-conditioned. We show how the problem may be reconditioned.

- From noisy f_k samples, retrieval of the correct value of n is difficult, and more so in case of clustered ϕ_j. Here, decimation of the data offers a way to obtain a reliable estimate of n automatically.

- Such decimation allows to divide and conquer the inverse problem statement. The smaller subproblems are largely independent and can be solved in parallel, leading to an improved complexity and efficiency.

- At the same time, the sub-Nyquist Prony method proves to be robust with respect to outliers in the data. Making use of some approximation theory results [9, 10], we can also validate the computation of the ϕ_j and α_j.

- The Nyquist constraint effectively restricts the bandwidth of the $\Im(\phi_j)$. Therefore, avoiding the constraint offers so-called superresolution, or the possibility to unearth higher frequency components in the samples.

All of the above can be generalized in several ways, to the use of more functions besides the exponential on the one hand, and to the solution of multdimensional inverse problems as in (1) on the other [5].

References

[1] M. Briani, A. Cuyt, F. Knaepkens, and W.-s. Lee. VEXPA: Validated EXPonential Analysis through regular subsampling. Signal Processing, 2020.

[2] Emmanuel J. Candès, Justin Romberg, and Terence Tao. Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information. *IEEE Trans. Inf. Theory*, 52(2):489–509, 2006.

[3] Annie Cuyt and Wen-shin Lee. Smart data sampling and data reconstruction. US Patent 9,690,749.

[4] Annie Cuyt and Wen-shin Lee. Smart data sampling and data reconstruction. EP2745404B1.

[5] Annie Cuyt and Wen-shin Lee. Multivariate exponential analysis from the minimal number of samples. *Adv. Comput. Math.,* 44:987–1002, 2018.

[6] Annie Cuyt and Wen-shin Lee. How to get high resolution results from sparse and coarsely sampled data. *Appl. Comput. Harmon. Anal.*, 48:1066–1087, 2020.

[7] R. de Prony. Essai expérimental et analytique sur les lois de la dilatabilité des fluides élastiques et sur celles de la force expansive de la vapeur de l'eau et de la vapeur de l'alkool, à différentes températures. *J. Ec. Poly.*, 1(22):24–76, 1795.

[8] D. L. Donoho. Compressed sensing. *IEEE Transactions on Information Theory*, 52(4):1289–1306, 2006.

[9] J. Gilewicz and M. Pindor. Padé approximants and noise: a case of geometric series. *J. Comput. Appl. Math.*, 87:199–214, 1997.

[10] J. Gilewicz and M. Pindor. Padé approximants and noise: rational functions. *J. Comput. Appl. Math.*, 105:285–297, 1999.

[11] H. Nyquist. Certain topics in telegraph transmission theory. *Trans. Am. Inst. Electr. Eng.*, 47(2):617–644, April 1928.

`annie.cuyt@uantwerpen.be`

Phase Retrieval Using Deep Auto-Decoders

Shai Dekel (Tel-Aviv University), Leon Gugel*

Phase retrieval is a well known ill-posed inverse problem where one tries to recover images given only the magnitude values of their Fourier transform as input. In recent years, new algorithms based on deep learning have been proposed, providing breakthrough results that surpass the results of the classical methods. In this work we provide a novel deep learning architecture PR-DAD (Phase Retrieval Using Deep Auto-Decoders), whose components are carefully designed based on sparsity modeling of the phase retrieval problem.

`shaidekel6@gmail.com`

Decimated Prony's Method for Stable Super-resolution

Nuha Diab (Tel Aviv University), Rami Katz and Dmitry Batenkov*

We study recovery of amplitudes and nodes of a finite impulse train from a limited number of equispaced noisy frequency samples. This problem is known as super-resolution (SR) under sparsity constraints and has numerous applications, including direction of arrival and finite rate of innovation sampling. Prony's method is an algebraic technique which fully recovers the signal parameters in the absence of measurement noise. In the presence of noise, Prony's method may experience significant loss of accuracy, especially when the separation between Dirac pulses is smaller than the Nyquist-Shannon-Rayleigh (NSR) limit. In this work we combine Prony's method with a recently established decimation technique for analyzing the SR problem in the regime where the distance between two or more pulses is much smaller than the NSR limit. We show that our approach attains optimal asymptotic stability in the presence of noise. Our result challenges the conventional belief that Prony-type methods tend to be highly numerically unstable.

`nuhadiab@tauex.tau.ac.il`

Integration on Subdivision Surfaces

Alexander Dietz (Technical University Darmstadt)

Subdivision surfaces can be used for simulation in the context of the Isogeometric Analysis. However, for the assembly of the Galerkin system it is necessary to compute numerically integrals on such subdivision surfaces. Since errors in the integration impair the accuracy of simulation, these integrals should be calculated with high precision. In this talk we discuss strategies for that purpose. In particular, we propose an adaptive scheme, which is efficient and reliably produces results of prescribed accuracy, both on regular and extraordinary parts of the surface.

Further, we present first experimental results and demonstrate how the accuracy of quadrature affects the accuracy of the simulation.

`dietz@mathematik.tu-darmstadt.de`

Multivariate C^∞ compactly supported basic limit functions

Nira Dyn (Tel Aviv University, Israel), Maria Charina, Costanza Conti*

In this talk we discuss the generation of multivariate C^∞ functions with compact small aupports by non-stationary subdivision schemes. Following the construction of such a univariate function, called "Up function", by a univariate non-stationary scheme, based on the masks of stationary schemes generating B-splines of growing degrees, we term the multivariate functions we generate "Up-like functions". We generate them by non-stationary schemes based on the masks of stationary schemes generating box-splines of growing supports.

We also present new analysis tools for proving the convergence and smoothness of certain classes of non-stationary schemes, classes which are wider than those generating the Up-like functions. A method for decreasing the supports of the Up-like functions is also discussed. For example, in the univariate case, we can get by this method a C^∞ functions with support $[0, 1+\epsilon]$, with ϵ arbitrarily small, in comparison with the support $[0, 2]$ of tht Up-function. We call these functions also Up-like functions. Examples of univariate and bivariate Up-like functions will be presented.

`niradyn@tauex.tau.ac.il`

Phase Retrieval from Spectrogram Measurements

Frank Filbir (Helmholtz Center & Technical University of Munich)

Phase retrieval (PR) in general refers to the problem of recovering a signal f from the magnitude of its frame coefficients $|\langle f, \phi_m \rangle|$. In the classical setting the analysing frame ϕ_m is given by the Fourier basis. The PR problem is of great interest in various fields of applied science like crystallography, diffraction imaging and many more. Recently a new diffraction imaging technique, now known as ptychographical imaging, was developed in order to image materials and biological tissue. Ptychography is a purely computational imaging technique. A detector (CCD camera) measures the intensity of many diffraction patterns each obtained by illuminating a small part of the object at a time. The measurements are produced by using light (X- rays) of one specific very short wavelength λ or an electron beam. The detector placed in the far-field distance (Fraunhofer diffraction). Mathematically this experimental set-up leads to the problem of phase retrieval from spectrogram measurements. That means we are given samples of

$$\mathfrak{I}(x,\xi) = \Big| \int_{\mathbb{R}^2} f(t)\, g(t-x)\, e^{-2\pi i\, \xi\cdot t}\, dt \Big|^2$$

for a known window function g. The aim is to reconstruct the object f. However, often experimental set-ups do not allow to work with one specific wavelength λ but we have to deal with polychromatic measurements, i.e. we are given

$$\mathfrak{I}_\lambda(x,\xi) = \Big| \int_{\mathbb{R}^2} f(t)\, g_\lambda(t-x)\, e^{-2\pi i\, \xi\cdot t/\lambda}\, dt \Big|^2$$

for $\lambda \in \{\lambda_1, \dots, \lambda_L\}$. Moreover, in many cases even the g_λ is unknown. This leads to what is called *Blind Polychromatic Ptychographic Imaging* (BPPI). In this talk we will provide an overview of BPPI and we present some reconstruction methods and results. The talk is based on joint work with Oleh Melnyk (Helmholtz Munich & Technische Universität München), Jan Rothardt (GSI Jena), Nico Hoffmann (HZDR, Dresden) within the project `AsoftXm` funded by the Helmholtz Imaging Platform (`HIP`), and the group of Christian Schroer (DESY, Hamburg) within the project `Ptychography4.0`.

`frank_filbir@web.de`

Super-smoothness of C^1 cubic Powell–Sabin splines

Jan Grošelj (University of Ljubljana), Hendrik Speleers*

The characterization of C^1 cubic splines over a general triangulation has proven to be difficult as geometry has a substantial impact on the number of degrees of freedom. However, if the triangulation is refined in a certain way, the problem becomes much more attainable. The use of the prominent Powell–Sabin 6-split technique allows not only the characterization of C^1 cubic splines over the refinement but also the construction of a B-spline-like basis that possesses a number of important properties for numerical analysis, namely, stability, local support, partition of unity, and non-negativity [1].

In this contribution, we will investigate the dual basis representation for the C^1 cubic Powell–Sabin B-splines in terms of the blossoming operator with the aim to extract conditions for C^2 super-smoothness. It turns out that some of these conditions arise very naturally, while others require specific geometric properties to be fulfilled [2,3]. We will demonstrate how C^2 super-smoothness can be exploited over partially structured triangulations to reduce the number of degrees of freedom of a spline without compromising its optimal approximation power.

References

[1] J. Grošelj and H. Speleers. *Construction and analysis of cubic Powell–Sabin B-splines.* Comput. Aided Geom. Design 57, 1–22, 2017.

[2] J. Grošelj and H. Speleers. *Three recipes for quasi-interpolation with cubic Powell–Sabin splines.* Comput. Aided Geom. Design 67, 47–70, 2018.

[3] J. Grošelj and H. Speleers. *Super-smooth cubic Powell–Sabin splines on three-directional triangulations: B-spline representation and subdivision.* J. Comput. Appl. Math. 386, 23 pages, 2021.

`jan.groselj@fmf.uni-lj.si`

Jackson and Bernstein inequalities for RBF interpolation in high order Sobolev spaces

Thomas Hangelbroek (University of Hawai'i), Christian Rieger (Philipps-Universität Marburg)*

Standard error estimates for interpolation with radial basis functions (RBFs) provide convergence rates if the error is measured in a low smoothness norm. As one might expect, these rates deteriorate if the error is measured in stronger norms and improve for target functions with greater regularity, up to a saturation class consisting of functions which are twice as smooth as required by the reproducing kernel Hilbert space associated with the RBF. This is explained by the so-called doubling trick. In this talk we will extend doubling results to the case where the error is measured in norms which are stronger than the so-called native space norm. As a by-product, we will prove new Bernstein estimates in this setting. Extensions to kernel interpolation on compact Riemannian manifolds will also be discussed.

`hangelbr@math.hawaii.edu`

Compensation of part deviations in additive manufacturing

Florian Heinrich (University of Passau)

Casting highly complex metal parts requires geometrically complex sand cores that cannot be produced by conventional core shooting. A more and more common solution is the additive manufacturing of sand cores that allow for more geometrical flexibility. As a downside the process is more complex and less stable so that produced cores deviate from their intended geometry.

In this talk we give a possible solution to this problem by printing the sand core first, measuring the deviations and deforming the CAD model to compensate the process induced deviations.

`florian.heinrich@uni-passau.de`

Application of spline based versions of SEIR model to Covid-19 spread

Ognyan Kounchev (Bulgarian Academy of Sciences)

The Covid-19 pandemic has caused an outbreak of interest to more refined mathematical methods of modeling the spread of the disease. Among the most used classical methods one has to mention the so-called Compartmental deterministic methods and one of its main representatives, SIR or more generally SEIR. This model represents a system of Ordinary Differential Equations which describes the evolution of the main variables of the Covid-19 spread. The variable S denotes the number of susceptibles, E denotes the number of exposed persons, I denotes the number of infected, and finally, R denotes the number of "removed" cases (includes the recovered and the deceased). The system depends on two important parameters $\beta(t)$ and $\gamma(t)$, where β stands for "transmission rate, which reflects the virus spread by infected individuals", and γ stands for "rate of hospitalization/isolation measures plus fatalities".

One of the main purposes of applying such models is to assess how the expensive restriction measures imposed by the authorities (home and social isolation/quarantine, travel restrictions, etc.) can effectively reduce the basic reproduction number of the disease and its transmission risk. In particular, it is essential to assess how the expensive, resource-intensive measures implemented by the authorities can contribute to the prevention and control of the COVID-19 infection, and how long they should be maintained.

The classical SIR/SEIR models have been primarily studied with constant rates β and γ, which does not reflect in a proper way their extremely dynamical behavior during the COVID-19 epidemic, especially the behavior resulting from the imposition of intensive restriction measures by the authorities.

We have proposed a very natural time-varying model for the parameters $\beta(t)$ and $\gamma(t)$ based on splines, which may be used for the analysis of historical data as well as for forecasting of the pandemic.

We discuss different aspects of the applications of these models to real life data for the Covid-19 spread, in particular the number of knots and their allocation for achieving better fitting to the data.

References

[1] Kounchev, Ognyan, Georgi Simeonov, and Zhana Kuncheva. "Scenarios for the spread of COVID-19 analyzed by the TVBG-SEIR spline model." Biomath 10.1 (2021): ID-2103087.

[2] Kounchev, Ognyan, Georgi Simeonov, and Zhana Kuncheva. "Estimation of the duration of covid-19 epidemic in a single country, with or without vaccinations. the case of bulgaria and germany." Comptes Rendus de l'Academie Bulgare des Sciences (2021): 677-686.

[3] Argirova, R. et al. "A nationwide analysis of the impact of COVID-19 in patients with a cardiovascular, oncological or chronic pulmonarydisease in the context of an eastern European country with a low vaccination rate: Bulgaria, March 2020 - April 2022", accepted in BMJ Open, 2023.

`kounchev@abv.bg`

Stable mapped polynomial interpolation through (β, γ)-Chebyshev points

Francesco Marchetti (University of Padova), Stefano De Marchi, Giacomo Elefante*

Using the so-called mapped bases or Fake Nodes Approach (FNA), it is possible to obtain a precise mapped polynomial interpolant that is not affected by the Runge's phenomenon by mapping the equispaced nodes onto the Chebyshev-Lobatto nodes in $[-1, 1]$. Similarly, discontinuous functions can be recovered avoiding the effects of the Gibbs phenomenon by choosing a suitable mapping. In order to design a scheme to prevent the appearance of both Runge's and Gibbs phenomena, we need to study and then employ a family of generalized Chebyshev points, i.e., (β, γ)-Chebyshev points, which are associated to a family of functions that are orthogonal in a subset of $[-1, 1]$. In this talk, we present various obtained results and ongoing research on these topics.

`francesco.marchetti@unipd.it`

Numerical approximation for image reconstruction in the 'Spectrometer/Telescope for Imaging X-rays (STIX)' mission on-board 'Solar Orbiter'

Michele Piana (University of Genova and Istituto Nazionale di Astrofisica, Osservatorio Astrofisico di Torino), Paolo Massa (Department of Physics and Astronomy, Western Kentucky University), Anna Volpara (Dipartimento di Matematica, Università di Genova), Sara Garbarino (Dipartimento di Matematica, Università di Genova), Federico Benvenuto (Dipartimento di Matematica, Università di Genova), Emma Perracchione (Dipartimento di Scienze Matematiche, Politecnico di Torino), Anna Maria Massone (Dipartimento di Matematica, Università di Genova)*

The 'Spectrometer/Telescope for Imaging X-rays (STIX)' on-board 'Solar Orbiter' modulates the incident radiation by means of sub-collimators that provide information on the complex values of specific Fourier components of the flaring X-ray source. This fact implies that the image reconstruction problem for STIX requires the solution of an inverse Fourier transform problem from limited data made of two steps: the interpolation of the recorded signal in the spatial frequency domain, and the out-of-band extrapolation of the interpolated data to obtain an appropriate spatial resolution in the image domain. This talk will illustrate the STIX image formation model and discuss some image reconstruction methods applied to the first experimental observations provided by this Solar Orbiter mission.

`piana@dima.unige.it`

Spline Representation and Redundancies of One-Dimensional ReLU Neural Network Models

Gerlind Plonka (University of Göttingen), Yannick Riebe, Yurii Kolomoitsev*

In this talk, we analyze the structure of a one-dimensional deep ReLU neural network (ReLU DNN) in comparison to the model of continuous piecewise linear (CPL) spline functions with arbitrary knots. In particular, we give a recursive algorithm to transfer the parameter set determining the ReLU DNN into the parameter set of a CPL spline function. Using this representation, we show that after removing the well-known parameter redundancies of the ReLU DNN, which are caused by the positive scaling property, all remaining parameters are independent. Moreover, we show that the ReLU DNN with one, two or three hidden layers can represent CPL spline functions with K arbitrarily prescribed knots (breakpoints), where K is the number of real parameters determining the normalized ReLU DNN (up to the output layer parameters). Our findings are useful to fix a priori conditions on the ReLU DNN to achieve an output with prescribed breakpoints and function values.

`plonka@math.uni-goettingen.de`

Prospects of Subdivision in Isogeometric Analysis

Ulrich Reif (Technical University of Darmstadt)

Due to built-in refinability, subdivision algorithms are a promising tool for the construction of function spaces in the context of Isogeometric Analysis. In this talk, we review scope and limitations of known theory with a special focus on the trivariate case, which is of paramount importance for applications. We come to the conclusion that our understanding of several crucial aspects is insufficient. This concerns basic tasks like the construction of appropriate algorithms, theoretical challenges like a regularity and convergence analysis, as well as practical issues like accurate numerical integration. Our considerations result in a list of steps that have to be taken to lay solid ground for the use of subdivision methods in simulation.

`reif@mathematik.tu-darmstadt.de`

Designing an algorithm for low-dose Poisson phase retrieval

Patricia Römer (Technical University of Munich and Helmholtz Center Munich), Benedikt Diederichs (Helmholtz Center Munich), Frank Filbir (Helmholtz Center Munich)*

Many experiments in the field of optical imaging are modelled as phase retrieval problems. Motivated by imaging experiments with biological specimens that need to be measured using a preferably low dose of illumination particles, we consider phase retrieval systems with small measurements. In such a setting, Poisson noise plays the dominant role. In this talk, we discuss how to formulate a suitable optimization problem. We study reasonable loss functions adapted to the Poisson distribution, optimized for low-dose data. As a solver, we apply Wirtinger flow type algorithms. For all proposed loss functions, we analyze the convergence of the respective Wirtinger flow type algorithms to stationary points. We present numerical reconstructions from phase retrieval measurements in a low-dose regime to corroborate our theoretical observations.

`patricia.roemer@tum.de`

Exact sphere representations over Platonic solids based on rational multisided Bézier patches

Ada Šadl Praprotnik (Institute of Mathematics, Physics and Mechanics, Slovenia), Jan Grošelj*

In this talk, we use rational multisided Bézier patches to obtain exact sphere representations over Platonic solids inscribed into the sphere [3].

First, we recall a class of multisided Bézier patches, the so-called S-patches [4], which unify and generalize triangular and tensor product Bézier patches. An S-patch can be defined over any convex n-sided polygon, $n \geq 3$, and is obtained by first embedding the polygon into the simplex of dimension $n-1$ and then defining the multivariate rational Bézier patch of degree d over the simplex. The 3-sided S-patches correspond to triangular Bézier patches and tensor product Bézier patches are a special case of 4-sided S-patches.

Using the framework of S-patches, we develop a general method for exact representation of a sphere section over the Platonic solids that utilizes the (inverse) stereographic projection. We apply the method to the faces of all five Platonic solids inscribed into the sphere. Depending on the Platonic solid, the obtained S-patches are defined over triangular, square, or pentagonal domains. This approach unifies two previously known constructions based on triangular and tensor product Bézier patches [1, 2] and introduces three new patches that together enable the representation of the sphere over all five Platonic solids. We show how the obtained sphere section representations can be used to cover the whole sphere.

References

[1] J. E. Cobb, Tiling the sphere with rational Bézier patches, *University of Utah Computer Science Technical Report*, UUCS-88-009, 1–14, 1988.

[2] G. Farin, B. Piper and A. J. Worsey, The octant of a sphere as a non-degenerate triangular Bézier patch, *Computer Aided Geometric Design*, Volume 4, 329–332, 1987.

[3] J. Grošelj and A. Šadl Praprotnik, Exact sphere representations over Platonic solids based on rational multisided Bézier patches, *Computer Aided Geometric Design*, Volume 98, 2022.

[4] C. T. Loop and T. D. DeRose, A multisided generalization of Bézier surfaces, *ACM Trans. Graph.*, Volume 8, 204–234, 1989.

`ada.sadl-praprotnik@imfm.si`

The Definite Moment for Multivariate Continued Fractions

Tomas Sauer (University of Passau and Fraunhofer IIS), Yuan Xu*

In a single variable, moment sequences, Gauss quadrature and continued fractions are closely related; in fact, many of the concepts turn out to be equivalent. In several variables, the situation is significantly different: there is no concept of continued fractions, orthogonal polynomials exist, but are more intricate, three-term recurrences have a complex structure and the existence of Gaussian cubature is a rarity.

By relating all these concepts and intertwining them with Prony's problem, we come up with a concept of multivariate continued fractions as rational approximations of formal Laurent series that relate Gaussian quadrature to the flat extension of moment sequences.

`Tomas.Sauer@uni-passau.de`

Learning Filters and Wavelets

Kathrin Schiermeier (University of Passau)

Filterbanks can be considered as simple neural networks, where the filters take the role of the weights that can be learned. So it is possible to learn filters with certain properties fitting for given data, for example scaling and wavelet filters. Most of the required properties are easy to implement, but one of them takes a little more effort: The Cohen criterion. The talk will show a possibility to deal with this difficulty using the Bernstein inequality.
Another possibility to use neural networks in the context of filterbanks is the completion of a perfect reconstruction filterbank when only one unimodular filter is given. In two dimensions this is a difficult task and although the perfect reconstruction is only learned for a certain training data set, it works surprisingly well for other (but similar) data.

`Kathrin.Schiermeier@uni-passau.de`

Maximally smooth splines on refined triangulations: Local simplex spline bases

Hendrik Speleers (University of Rome Tor Vergata), Tom Lyche and Carla Manni*

Splines on triangulations have widespread applications in many areas, ranging from finite element analysis to computer graphics. Highly smooth spline spaces are often preferred.

When dealing with a general triangulation, to obtain splines of high smoothness in a stable manner, sufficiently large degrees have to be considered. An alternative is to use lower-degree macro-elements that subdivide each triangle into a number of subtriangles (or more general subdomains).

Simplex splines are one of the most elegant generalizations of univariate B-splines to the multivariate setting. They enjoy a nice geometric interpretation and several properties such as smoothness and recursion, knot insertion and degree elevation formulas.

In this talk we consider a family of macro-elements of degree p and maximal smoothness $p-1$ on a triangular region and we discuss the construction of a suitable local representation for the related spline space in terms of simplex splines. In particular, we detail the important cases of C^2 cubic [1] and C^3 quartic macro-elements, and we discuss several properties, such as local support, linear independence, and nonnegative partition of unity of the provided simplex spline basis.

References

[1] T. Lyche, C. Manni, and H. Speleers. *Construction of C^2 cubic splines on arbitrary triangulations*. Foundations of Computational Mathematics 22, 1309–1350, 2022.

`speleers@mat.uniroma2.it`

Sparse Volume Reconstruction Based on Haar Wavelet Techniques

A. Michael Stock (University of Passau), Tomas Sauer*

The goal is to reconstruct 3D computed tomography (CT) volumes that are locally constant. By doing this in the Haar wavelet basis, we can save memory because the thresholding/compression step is integrated into the reconstruction process that uses classical iterative methods.

The main idea is to define a multilevel reconstruction algorithm where only the relevant coefficients are kept before advancing to the next resolution level. This enables local refinement and leads to a monotone sequence of finer grids where the data is reconstructed. The reconstruction steps itself consist of applying standard methods, e.g., the Algebraic Reconstruction Technique (ART), onto the data on the current grid. Iterating through the resolution levels, the grid locally approaches the maximal resolution while being sparse. First results on synthetic and real data look promising.

`stock@forwiss.uni-passau.de`

Optimal approximation of spherical squares by tensor product quadratic Bézier patches

Aleš Vavpetič (University of Ljubljana), Emil Žagar*

The main topic of this talk is the optimal approximation of spherical squares by tensor product quadratic Bézier patches. The measure of the quality is the radial error. An algorithm for the derivation of the unique optimal approximant is described, and the results are used for the construction of the continuous spline of six patches approximating the whole sphere. Furthermore, a generalization of the problem to spherical rectangles is considered. Several numerical examples reveal that this is quite a challenge since uncountably many optimal approximants might exist.

`ales.vavpetic@fmf.uni-lj.si`

Interpolation of planar G^1 data by Pythagorean-hodograph cubic biarcs with prescribed arc lengths

Emil Žagar (University of Ljubljana and Institute of mathematics, physics and mechanics, Ljubljana), Grega Cigler*

The interpolation of two points and two tangent directions by planar parametric cubic curves with prescribed arc lengths will be considered. It is well known that this problem is highly nonlinear if standard cubic curves are used. However, if Pythagorean-hodograph (PH) curves are considered, the problem simplifies due to their distinguished property that the arc length is a polynomial function of its coefficients. Since a single segment of a PH cubic curve does not provide enough free parameters, the so called PH cubic biarcs will be used. The lookup table of the solutions will be given enabling an easy implementation of the described method. Some quantities arising from geometric properties of the resulting curves will suggest the most appropriate one. Several numerical examples will be given together with an example of approximation of an analytic curve by G^1 PH cubic biarc spline curve. Finally, numerical estimation of the approximation order will be presented.

`emil.zagar@fmf.uni-lj.si`

List of participants

1. *Rosanna Campagna*, University of Campania “Luigi Vanvitelli”, `rosanna.campagna@unicampania.it`
2. *Mariantonia Cotronei*, University Mediterranea of Reggio Calabria, `mariantonia.cotronei@unirc.it`
3. *Annie Cuyt*, University of Antwerp & Shenzen University, `annie.cuyt@uantwerpen.be`
4. *Shai Dekel*, Tel-Aviv University, `shaidekel6@gmail.com`
5. *Nuha Diab*, Tel Aviv University, `nuhadiab@tauex.tau.ac.il`
6. *Alexander Dietz*, Technical University Darmstadt, `dietz@mathematik.tu-darmstadt.de`
7. *Nira Dyn*, Tel Aviv University, Israel, `niradyn@tauex.tau.ac.il`
8. *Frank Filbir*, Helmholtz Center & Technical University of Munich, `frank_filbir@web.de`
9. *Jan Grošelj*, University of Ljubljana, `jan.groselj@fmf.uni-lj.si`
10. *Thomas Hangelbroek*, University of Hawai‘i , `hangelbr@math.hawaii.edu`
11. *Florian Heinrich*, University of Passau, `florian.heinrich@uni-passau.de`
12. *Ognyan Kounchev*, Bulgarian Academy of Sciences, `kounchev@abv.bg`
13. *Francesco Marchetti*, University of Padova, `francesco.marchetti@unipd.it`
14. *Michele Piana*, University of Genova and Istituto Nazionale di Astrofisica, Osservatorio Astrofisico di Torino, `piana@dima.unige.it`
15. *Gerlind Plonka*, University of Göttingen, `plonka@math.uni-goettingen.de`
16. *Ulrich Reif*, Technical University of Darmstadt, `reif@mathematik.tu-darmstadt.de`
17. *Patricia Römer*, Technical University of Munich and Helmholtz Center Munich, `patricia.roemer@tum.de`
18. *Ada Šadl Praprotnik*, Institute of Mathematics, Physics and Mechanics, Slovenia, `ada.sadl-praprotnik@imfm.si`
19. *Tomas Sauer*, University of Passau and Fraunhofer IIS, `Tomas.Sauer@uni-passau.de`
20. *Kathrin Schiermeier*, University of Passau, `Kathrin.Schiermeier@uni-passau.de`
21. *Hendrik Speleers*, University of Rome Tor Vergata, `speleers@mat.uniroma2.it`
22. *A. Michael Stock*, University of Passau, `stock@forwiss.uni-passau.de`
23. *Aleš Vavpetič*, University of Ljubljana, `ales.vavpetic@fmf.uni-lj.si`
24. *Emil Žagar*, University of Ljubljana and Institute of mathematics, physics and mechanics, Ljubljana, `emil.zagar@fmf.uni-lj.si`