Self-adapting reproduction of trigonometric surfaces by non-linear subdivision

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IM-Workshop Bernried 2018

Definition (Subdivision scheme)

Given $f^0 \in l_\infty(\mathbb{Z}^n)$,

$$f^{k+1} := S^k f^k, \quad S^k : l_\infty(\mathbb{Z}^n) \longrightarrow l_\infty(\mathbb{Z}^n), \qquad k \geq 0.$$

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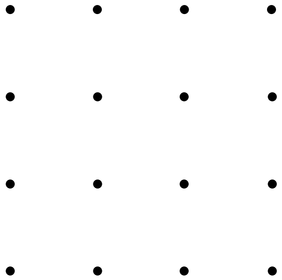

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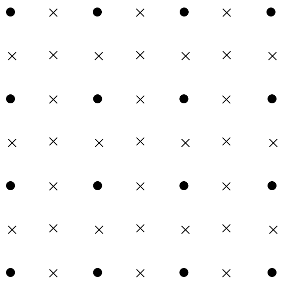

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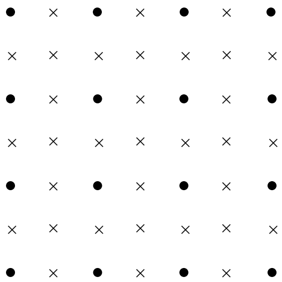

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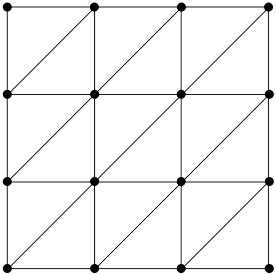

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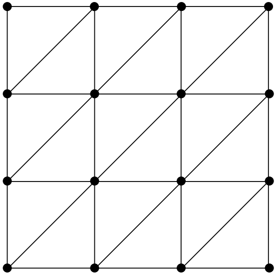

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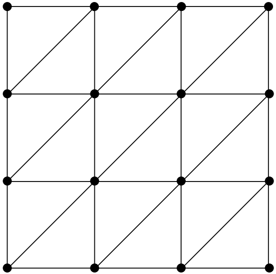

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- **Reproduction of trigonometric functions**

A subdivision scheme **reproduces** set of function V if for any $F \in V$

$$f^k = (F(\alpha 2^{-k}))_{\alpha\in\mathbb{Z}} \quad \Longrightarrow \quad f^{k+1} = (F(\alpha 2^{-(k+1)}))_{\alpha\in\mathbb{Z}}.$$

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Nira Dyn, David Levin and Ariel Luzzatto.
Exponentials reproducing subdivision schemes.
Foundations of Computational Mathematics, 3(2):187–206, 2003.

Costanza Conti and Lucia Romani.
Algebraic conditions on non-stationary subdivision symbols for exponential polynomial reproduction.
J. Comput. Appl. Math., 236(4):543–556, September 2011.

Exponential polynomials

$$V = \{\sum_{i=0}^{g}\sum_{n=0}^{\nu_i} c_{i,n} t^n \exp(\gamma_i t) : c_{i,n} \in \mathbb{R}\}.$$

$$\boxed{V_\gamma = \{\tilde{c}_{0,0}\cos(\gamma t) + \tilde{c}_{1,0}\sin(\gamma t) + \tilde{c}_{2,0} : \tilde{c}_{i,n} \in \mathbb{R}\}}$$

Linear non-stationary scheme:

$$f^{k+1}_{2\alpha+1} = \frac{1}{2}f^k_\alpha + \frac{1}{2}f^k_{\alpha+1} - \Gamma^k_\gamma\left(f^k_{\alpha+2} - f^k_{\alpha+1} - f^k_\alpha + f^k_{\alpha-1}\right)$$

$$\Gamma^k_\gamma = \frac{1}{2}\frac{1}{\left(2\sqrt{\frac{1+\cos(\gamma 2^{-k})}{2}}+1\right)^2 - 1}$$

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Rosa Donat, Sergio López-Ureña.
A non-linear stationary subdivision scheme that reproduces trigonometric functions.
In preparation.

Annihilation property

$$f^k_{\alpha-1} - (2\cos(\gamma 2^{-k}) + 1) f^k_{\alpha} + (2\cos(\gamma 2^{-k}) + 1) f^k_{\alpha+1} - f^k_{\alpha+2} = 0,$$

$$f^k_{\alpha} = F(\alpha 2^{-k})$$

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$$f^k_\alpha = F(\alpha 2^{-k}) \quad \Longrightarrow \quad \cos(\gamma 2^{-k}) = \frac{1}{2}\left(\frac{f^k_{\alpha+2} - f^k_{\alpha-1}}{f^k_{\alpha+1} - f^k_{\alpha}} - 1\right), \qquad \forall \in \mathbb{Z}.$$

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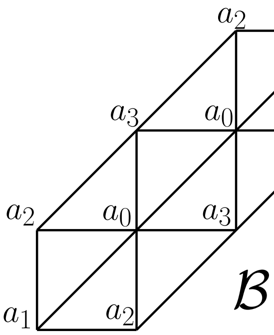
$$\Gamma^k_{\gamma} = \frac{1}{2}\frac{1}{\left(2\sqrt{\frac{1+\cos(\gamma 2^{-k})}{2}} + 1\right)^2 - 1} = \frac{1}{2}\frac{1}{\left(1 + \sqrt{1 + \frac{f^k_{\alpha+2} - f^k_{\alpha-1}}{f^k_{\alpha+1} - f^k_{\alpha}}}\right)^2 - 1}$$

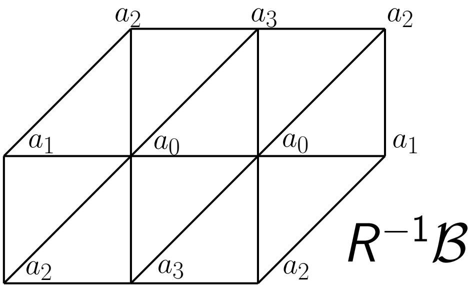
Non-linear stationary scheme

$$f^{k+1}_{2\alpha+1} = \frac{1}{2} f^k_{\alpha} + \frac{1}{2} f^k_{\alpha+1} - \frac{1}{2}\frac{1}{\left(1 + \sqrt{1 + \frac{f^k_{\alpha+2} - f^k_{\alpha-1}}{f^k_{\alpha+1} - f^k_{\alpha}}}\right)^2 - 1}\left(f^k_{\alpha+2} - f^k_{\alpha+1} - f^k_{\alpha} + f^k_{\alpha-1}\right)$$

$$f^k = \{f^k_\alpha, \alpha \in \mathbb{Z}^2\}$$

$$f^{k+1}_{2\alpha} = f^k_\alpha$$
$$f^{k+1}_{2\alpha+(1,1)} = \Psi^k((f^k_{\alpha+\beta})_{\beta\in\mathcal{B}})$$
$$f^{k+1}_{2\alpha+(1,0)} = \Psi^k((f^k_{\alpha+R^{-1}\beta})_{\beta\in\mathcal{B}})$$
$$f^{k+1}_{2\alpha+(0,1)} = \Psi^k((f^k_{\alpha+R\beta})_{\beta\in\mathcal{B}})$$





Notation

$$z = (z_1, z_2), \quad \gamma = (\gamma_1, \gamma_2), \quad \bar{\gamma} = (\gamma_1, -\gamma_2).$$

Space to reproduce

$$V_\gamma = \text{span}\left\{1, \exp(\pm\gamma \cdot z), \exp(\pm\bar{\gamma} \cdot z)\right\}.$$

which is

$$V_\gamma = \text{span}\left\{1, \cosh(\gamma_1 z_1 \pm \gamma_2 z_2), \sinh(\gamma_1 z_1 \pm \gamma_2 z_2)\right\},$$
$$V_{\imath\gamma} = \text{span}\left\{1, \cos(\gamma_1 z_1 \pm \gamma_2 z_2), \sin(\gamma_1 z_1 \pm \gamma_2 z_2)\right\}.$$

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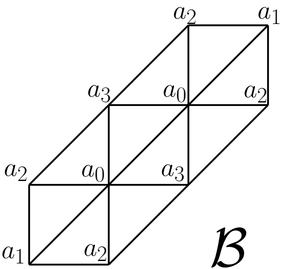
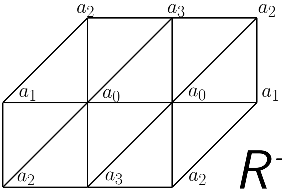


$\mathcal{B}$



$R^{-1}\mathcal{B}$

$$F(\Xi^{k+1}_{2\alpha+e_i}) = \Psi\left(\left(F(\Xi^{k}_{\alpha+R^i v})\right)_{v\in\mathcal{B}}\right), \quad i \in \{-1, 0, 1\}, \ \forall\alpha\in\mathbb{Z}^2, \ \forall F \in V_\gamma$$

$F \in V_\gamma \Longrightarrow F(\bullet + v) \in V_\gamma$

$\mathcal{B}$ and V_γ are symmetric respect to the axis $z_1 = z_2$

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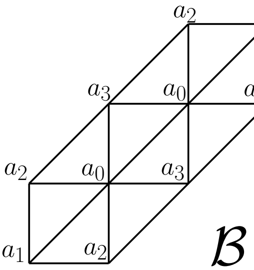
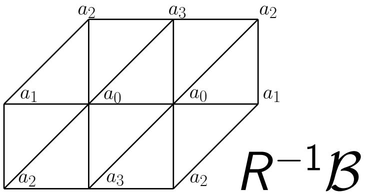
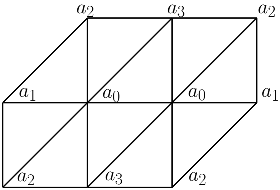




$$F(\Xi^{k+1}_{(1,1)}) = \Psi\left((F(\Xi^k_v))_{v\in\mathcal{B}}\right), \quad \forall F \in V_\gamma \text{ (diagonal edge)}$$

$$F(\Xi^{k+1}_{(1,0)}) = \Psi\left((F(\Xi^k_{R^{-1}v}))_{v\in\mathcal{B}}\right), \quad \forall F \in V_\gamma \text{ (horizontal edge)}$$

$$\xi_k := (\xi_{1,k}, \xi_{2,k}) := (\cosh(2^{-k}\gamma_1), \cosh(2^{-k}\gamma_2)), \qquad \xi_{j,k+1} = \sqrt{\frac{1+\xi_{j,k}}{2}}.$$



$$a_0^h = \frac{1}{2}\xi_{1,k+1}^{-1},$$
$$a_1^h = 0,$$
$$a_2^h = -\frac{1}{16}\xi_{1,k+1}^{-1}\xi_{1,k+2}^{-2},$$
$$a_3^h = \frac{1}{8}\xi_{1,k}\xi_{1,k+1}^{-1}\xi_{1,k+2}^{-2},$$

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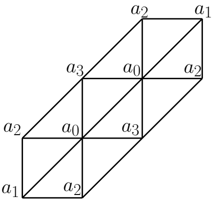

$$a_0^d = -\frac{\frac{1}{2}\xi_{1,k+1}^{-1}\xi_{2,k+1}^{-1}(\xi_{1,k}+\xi_{2,k}+2)+\xi_{1,k}+\xi_{2,k}}{2(\xi_{1,k}+\xi_{2,k}-2)},$$

$$a_1^d = 0,$$

$$a_2^d = \frac{\xi_{1,k+1}^{-1}\xi_{2,k+1}^{-1}-1}{4(\xi_{1,k}+\xi_{2,k}-2)},$$

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Theorem

$$\Delta_{v_1}^{\gamma_1} \Delta_{v_2}^{\gamma_2} \cdots \Delta_{v_m}^{\gamma_m} F = 0, \qquad \forall v_1, v_2, \ldots, v_m \in \mathbb{R}^2$$

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Annihilation of V_γ

$$\Delta^0_{2^{-k}v}\Delta^{\gamma}_{2^{-k}e_{-1}}\Delta^{-\gamma}_{2^{-k}e_{-1}}F = 0, \qquad \forall F \in V_\gamma,$$

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If the orientation is **horizontal**, the annihilation property leads to...

$$0 = f^k_{\alpha+(2,1)} - f^k_{\alpha+(1,0)} - 2\xi_{2,k}(f^k_{\alpha+(1,1)} - f^k_{\alpha}) + f^k_{\alpha+(0,1)} - f_{\alpha+(-1,0)}$$

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$$\Psi((f_v)_{v\in\mathcal{B}}) = \begin{cases} \Psi_h^{\xi_{1,k}}((f_v)_{v\in\mathcal{B}}), & \text{if unique solution for the 6 equations} \\ \Psi_d^{\xi_k}((f_v)_{v\in\mathcal{B}}), & \text{if not unique, but } f_\alpha \neq f_{\alpha+(1,1)} \\ \Psi_0((f_v)_{v\in\mathcal{B}}), & \text{otherwise} \end{cases}$$

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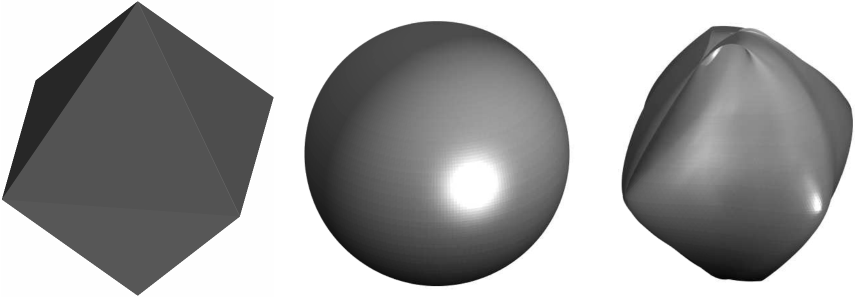
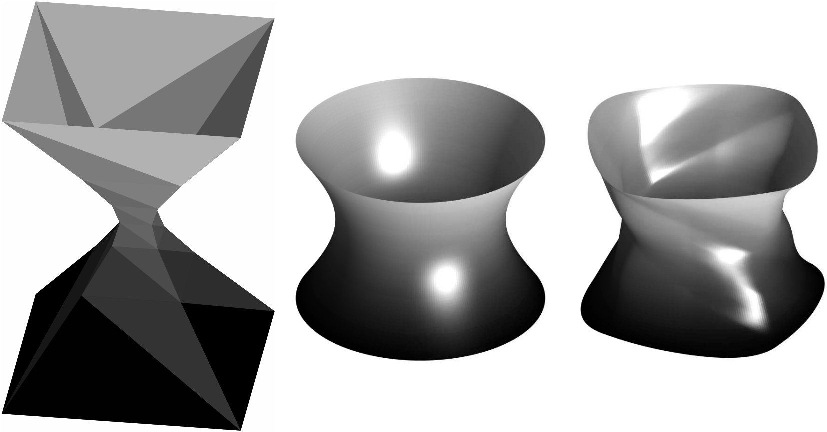
- If data does not come from V_γ, we use $\Psi_d^{\xi_k}((f_v)_{v\in\mathcal{B}})$ almost all the time
- We should be caution of $f_\alpha \neq f_{\alpha+(1,1)}$ in practice for reproducing V_γ

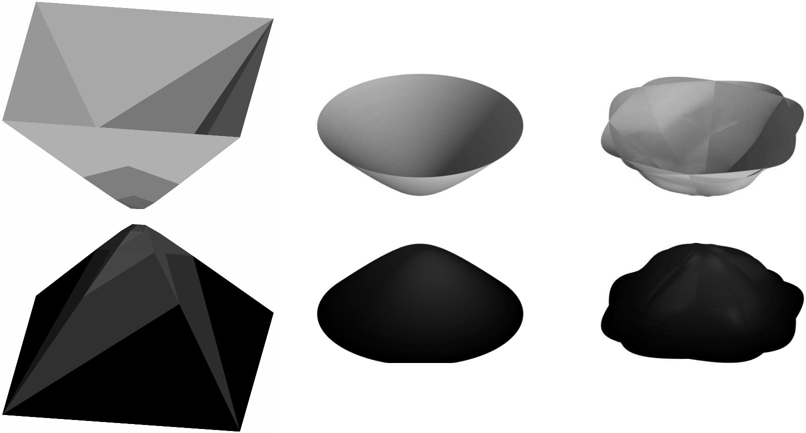


Sphere: $(u, v) \longmapsto (\sin(\pi/2v)\cos(\pi/2u), \sin(\pi/2v)\sin(\pi/2u), \cos(\pi/2v))$



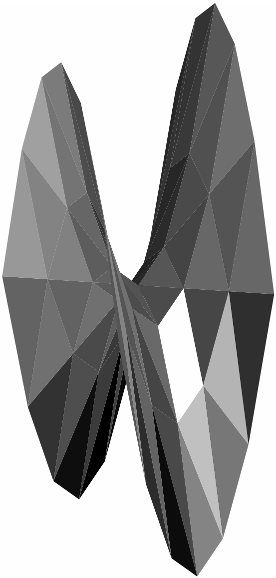
Hyperboloid:

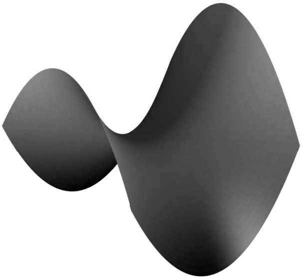
$$(u, v) \longmapsto (\cosh(9/10v)\cos(\pi/2u), \cosh(9/10v)\sin(\pi/2u), \sinh(9/10v))$$



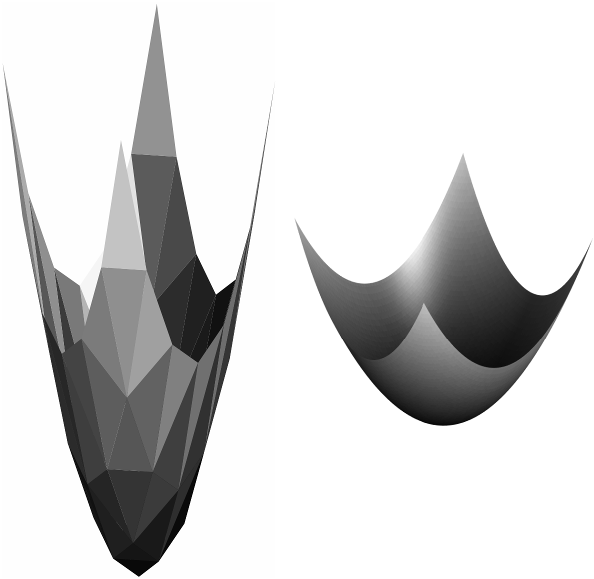
Elliptic hyperboloid:

$$(u, v) \longmapsto (\sinh(9/10v)\cos(\pi/2u), \sinh(9/10v)\sin(\pi/2u), \cosh(9/10v))$$

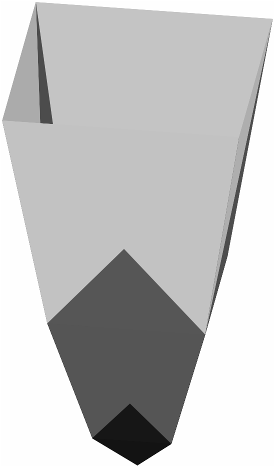
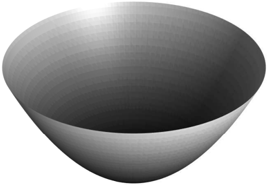
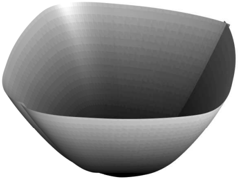




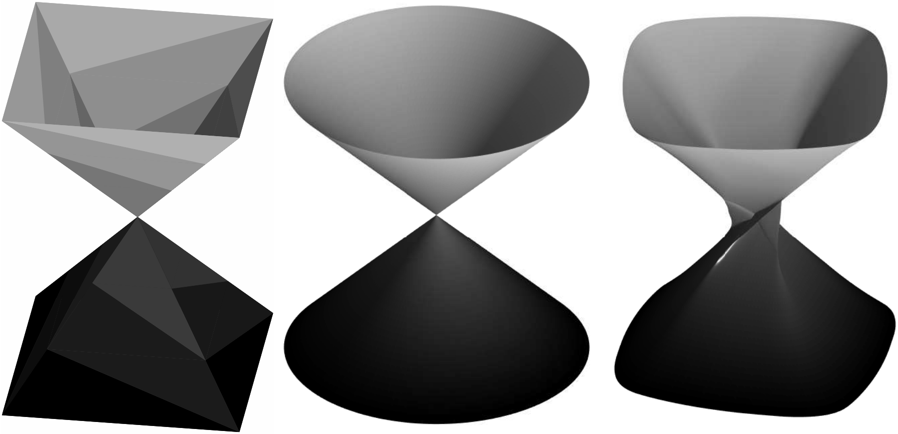
Hyperbolic paraboloid: $(u, v) \longmapsto (u, v, u^2 - v^2)$



Elliptic paraboloid: $(u, v) \longmapsto (u, v, u^2 + v^2)$

Elliptic paraboloid: $(u, v) \longmapsto (v\cos(u), v\sin(u), v^2)$



Cone: $(u, v) \longmapsto (v\cos(u), v\sin(u), v)$

Self-adapting reproduction of trigonometric surfaces by non-linear subdivision

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IM-Workshop Bernried 2018