

$$X_N(z) := \sum_{\nu=0}^N h_\nu z^{-\nu}, \quad |z| = 1,$$

$$\Psi'_N(\theta) := \frac{1}{2\pi} |X_N(e^{i\theta})|^2, \quad -\pi \leq \theta \leq \pi.$$

Moments for this distribution:

$$\begin{aligned} \mu_j &:= \int_{-\pi}^{\pi} e^{-ij\theta} d\Psi_N(\theta), \quad j \in \mathbb{Z}, \\ &= \begin{cases} \sum_{k=0}^{N-j-1} h_k \bar{h}_{k+j}, & j = 0, 1, 2, \dots, \\ \bar{\mu}_{-j}, & j = -1, -2, -3, \dots \end{cases} \\ \Delta_n &:= \det(\mu_{i-j})_{i,j=0,\dots,n} \end{aligned}$$

$$\tilde{\mathfrak{s}}_0(z) := 1, \quad \tilde{\mathfrak{s}}_0^*(z) := 1,$$

$$\tilde{\mathfrak{s}}_n(z) := \frac{1}{\Delta_{n-1}} \begin{vmatrix} \mu_0 & \cdots & \mu_{-n} \\ \vdots & & \vdots \\ \mu_{n-1} & \cdots & \mu_{-1} \\ 1 & \cdots & z^n \end{vmatrix},$$

$$\tilde{\mathfrak{s}}_n^*(z) := \frac{1}{\Delta_{n-1}} \begin{vmatrix} \mu_0 & \cdots & \mu_n \\ \vdots & & \vdots \\ \mu_{-n+1} & \cdots & \mu_1 \\ z^n & \cdots & 1 \end{vmatrix}, \quad n \geq 1,$$

$(\tilde{\mathfrak{s}}_\nu)_{\nu \in \mathbb{N}_0}$ monic, orthogonal with respect to

$$\langle f, g \rangle := \int_{-\pi}^{\pi} f(e^{i\theta}) \overline{g(e^{i\theta})} d\Psi_N(\theta)$$

$$\begin{aligned}
\tilde{s}_0(z) &:= \tilde{s}_0(z) := 1, \\
\tilde{s}_n(z) &= z\tilde{s}_{n-1}(z) + a_n\tilde{s}_{n-1}^*(z), \\
\tilde{s}_n^*(z) &= \bar{a}_nz\tilde{s}_n^*(z) + \tilde{s}_{n-1}(z), \quad n \geq 1.
\end{aligned}$$

$$\tilde{s}_{n-1}(z) =: \sum_{\nu=0}^{n-1} p_\nu^{(n-1)} z^\nu, \quad n \geq 1,$$

$$a_n = \tilde{s}_n(0) = -\frac{\langle z\tilde{s}_{n-1}, 1 \rangle}{\langle \tilde{s}_{n-1}^*, 1 \rangle} = -\frac{\sum_{\nu=0}^{n-1} p_\nu^{(n-1)} \mu_{-\nu-1}}{\sum_{\nu=0}^{n-1} p_\nu^{(n-1)} \mu_{\nu+1-n}}$$

$$\begin{aligned}
\delta_n &:= \langle \tilde{s}_n, \tilde{s}_n \rangle = \langle \tilde{s}_n^*, \tilde{s}_n^* \rangle \quad n \geq 0, \\
&= \delta_{n-1}(1 - |a_n|^2), \quad n \geq 1, \\
&= \mu_0 \prod_{k=1}^n (1 - |a_k|^2), \quad n \geq 0, \\
\langle \tilde{s}_n^*, z \tilde{s}_n \rangle &= \langle \tilde{s}_n^*, \tilde{s}_{n+1} - a_{n+1} \tilde{s}_n^* \rangle = -a_{n+1} \delta_n
\end{aligned}$$

Levinson Algorithm

$$\tilde{s}_0(z) := 1, \quad \delta_0 := \mu_0$$

For $n = 0, 1, 2, \dots$ do

$$a_{n+1} := -\frac{\langle z\tilde{s}_n, 1 \rangle}{\delta_n},$$

$$\tilde{s}_{n+1} := z\tilde{s}_n(z) + a_{n+1}\tilde{s}_n^*(z),$$

$$\delta_{n+1} := \delta_n(1 - |a_{n+1}|^2)$$

done

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