Prony' Idea and Approximation

Renormalized and overestimated degree of Prony polynomial

$$
\begin{aligned}
\rho_l(z) &:= \sum_{\nu=0}^{l} \tilde{p}_\nu z^\nu \,, \quad \tilde{p}_\nu = 1 \,, \quad \lfloor N/2 \rfloor \geq l \geq m \,, \\
\mathbf{H} &:= \left(h_{j+k}\right)_{\substack{j=0,\dots,N-l-1 \\ k=1,\dots,N-l}} \,, \qquad \mathbf{h} := \left(h_0, \dots, h_{N-l-1}\right)^T \,, \\
\tilde{\mathbf{p}} &:= \left(\tilde{p}_1, \dots, \tilde{p}_l\right)^T \\
\|z\|_\dagger &:= \|\operatorname{Re} z\|_1 + \|\operatorname{Im} z\|_1
\end{aligned}
$$

Problem: Overdetermined system

$$\mathbf{H}\tilde{\mathbf{p}} \;=\; -\mathbf{h}$$

Consider residuum

$$r(\tilde{\mathbf{p}}) := -\left(\mathbf{h} + \mathbf{H}\tilde{\mathbf{p}}\right)$$

Linear Optimization problem

$$\|r(\tilde{\mathbf{p}})\|_{\dagger} = \mathrm{Min.!}$$
$$\text{with respect to}$$
$$\mathbf{H}\tilde{\mathbf{p}} + r(\tilde{\mathbf{p}}) = -\mathbf{h}$$

Basic feasible solution $\tilde{\mathbf{p}} = \mathbf{0}, \; r(\tilde{\mathbf{p}}) = -\mathbf{h}$

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