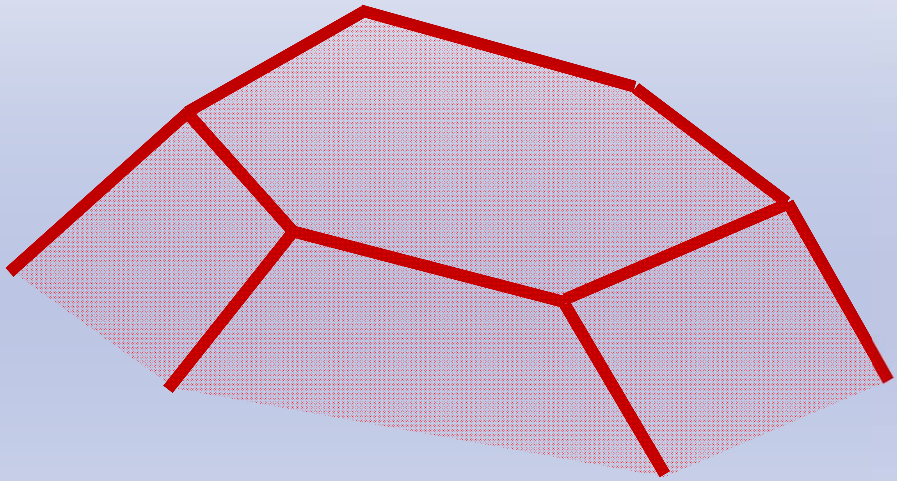
Mixed Primal Dual Honeycomb Schemes

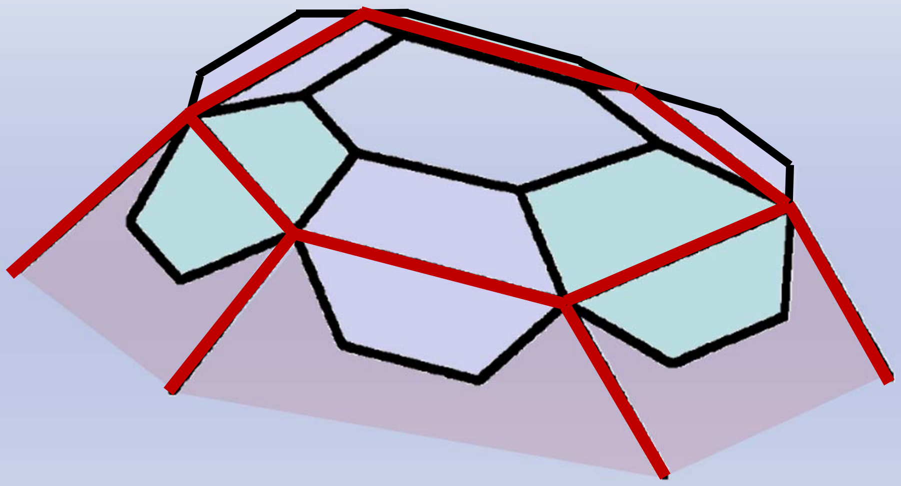
Hartmut Prautzsch
Huining Meng

Appl. Approx., Signals and Images, Bernried, 22.2.2018

The honeycomb scheme

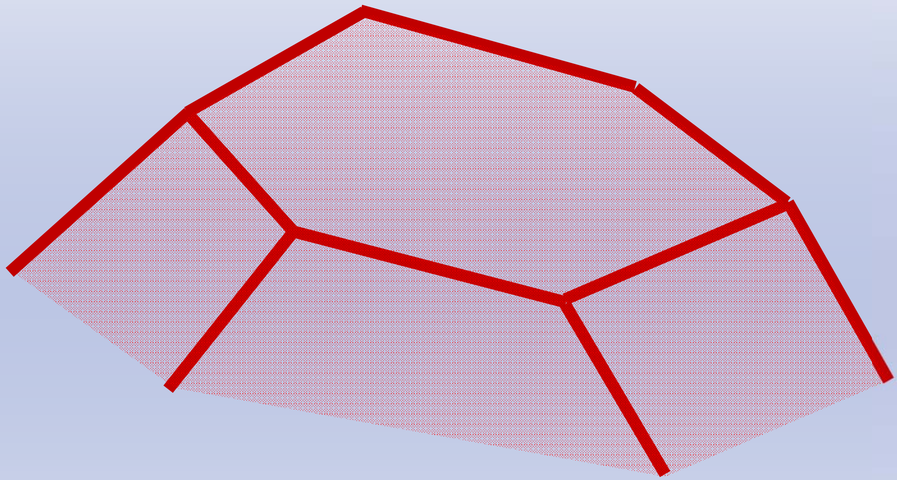
Dyn, Levin, Liu 1991

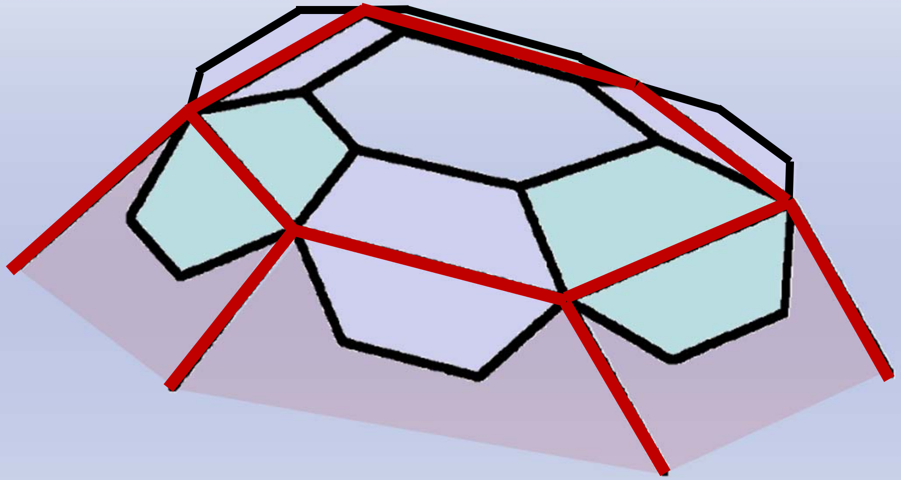




The honeycomb scheme
($\mathcal{H}$)

Dyn, Levin, Liu 1991





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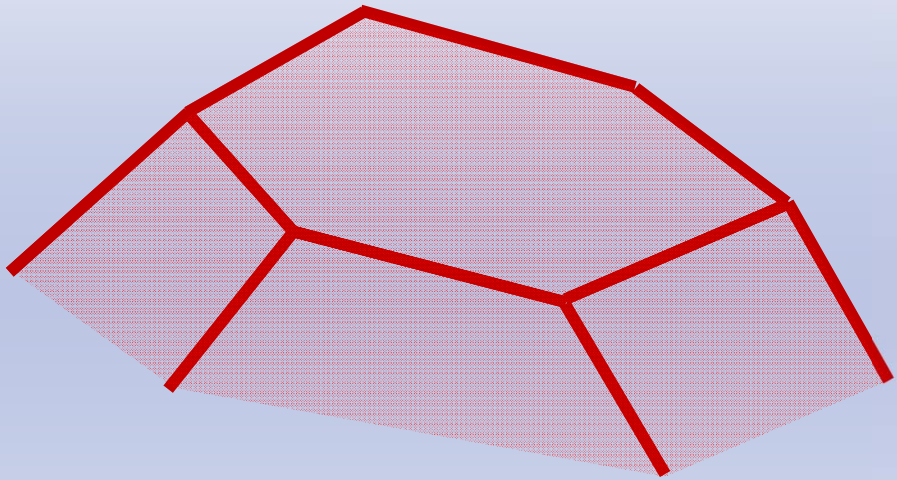
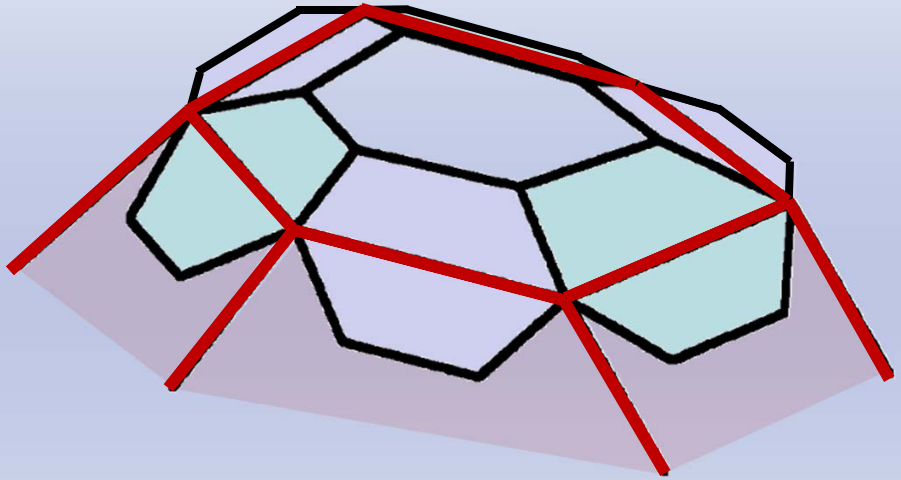
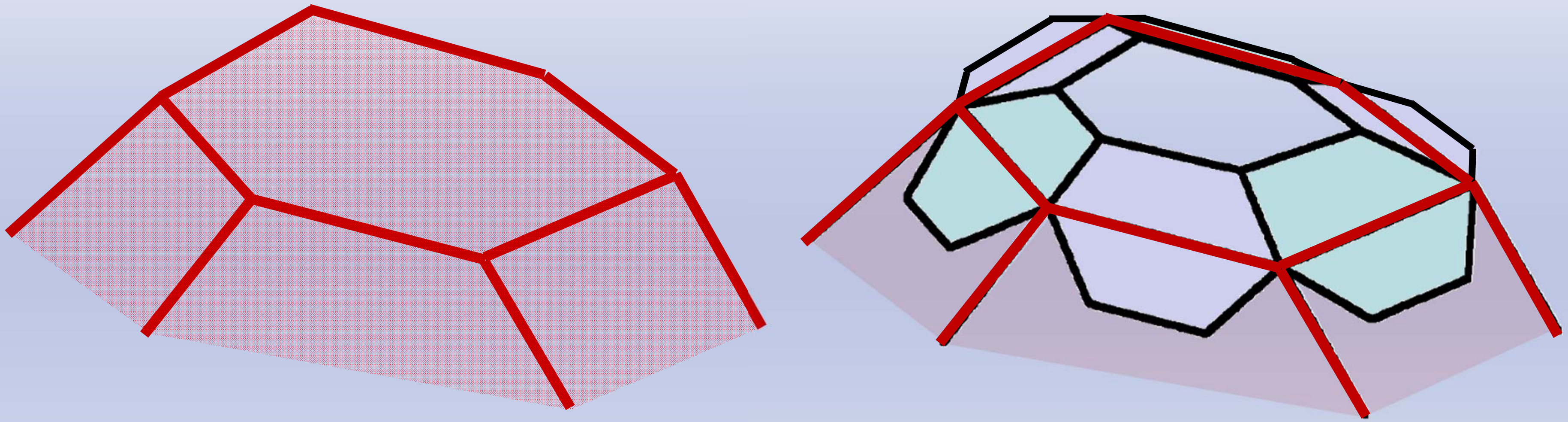




C^1 limits

The honeycomb scheme
($\mathcal{H}$)

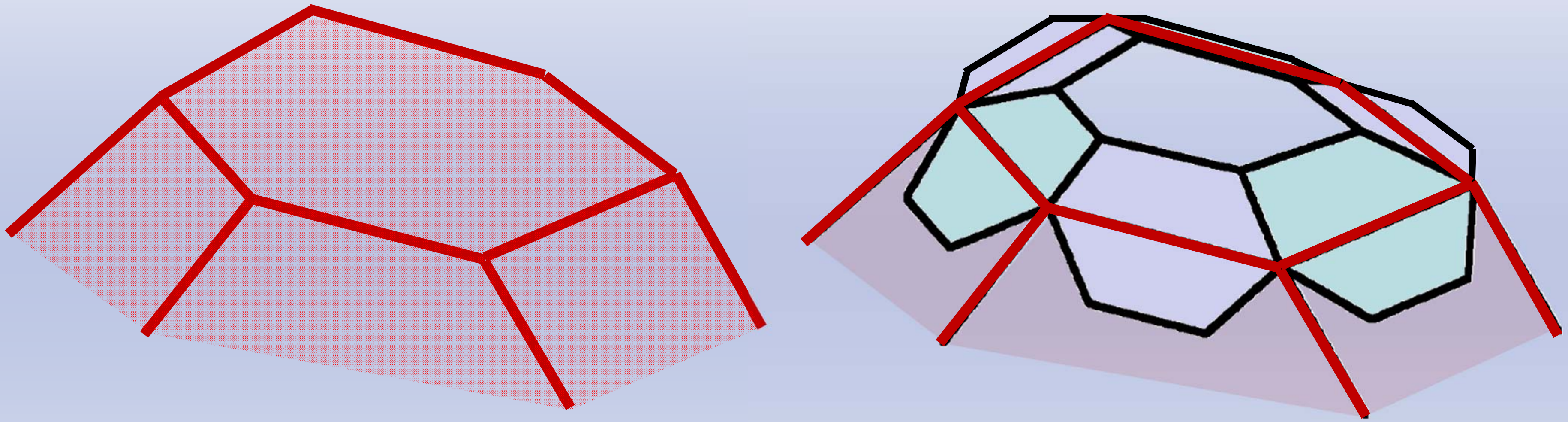
Dyn, Levin, Liu 1991



C[1] limits
interpolatory

The honeycomb scheme
($\mathcal{H}$)

Dyn, Levin, Liu 1991



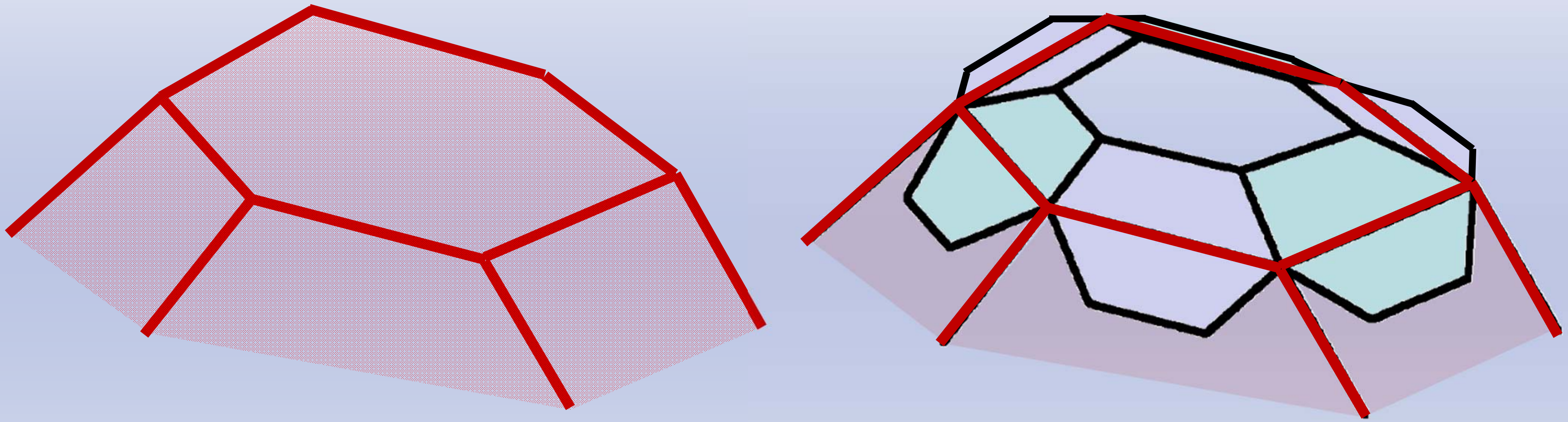
C^1 limits

interpolatory

convexity preserving

The honeycomb scheme
($\mathcal{H}$)

Dyn, Levin, Liu 1991



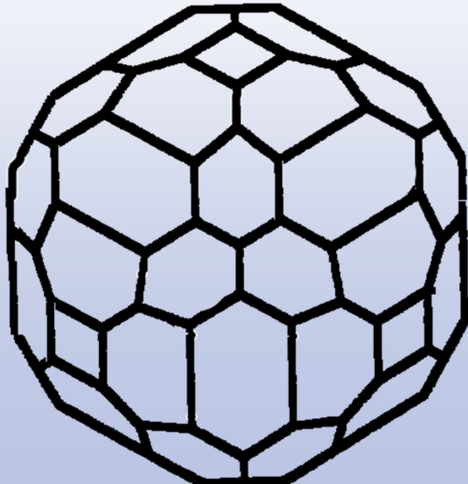
C^1 limits

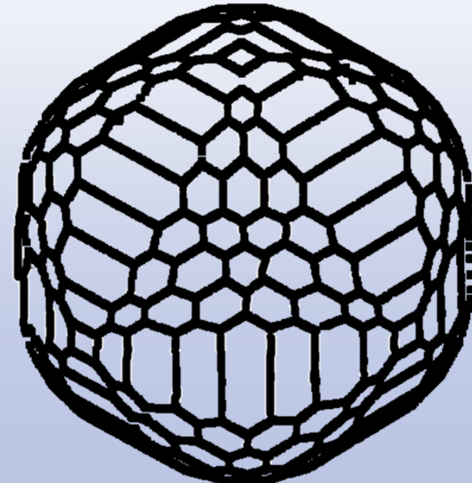
interpolatory

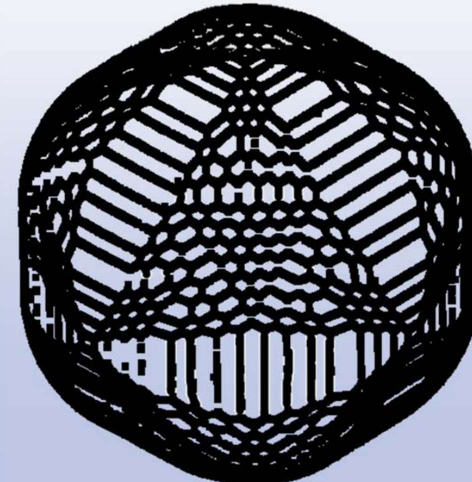
convexity preserving

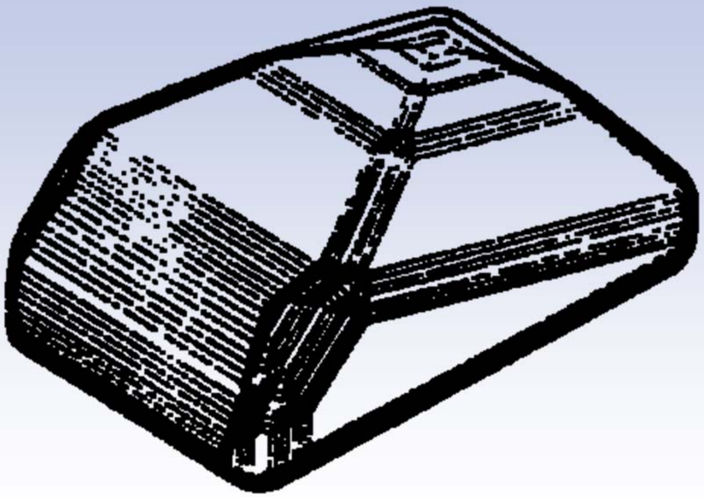
generalizable with enhanced proofs

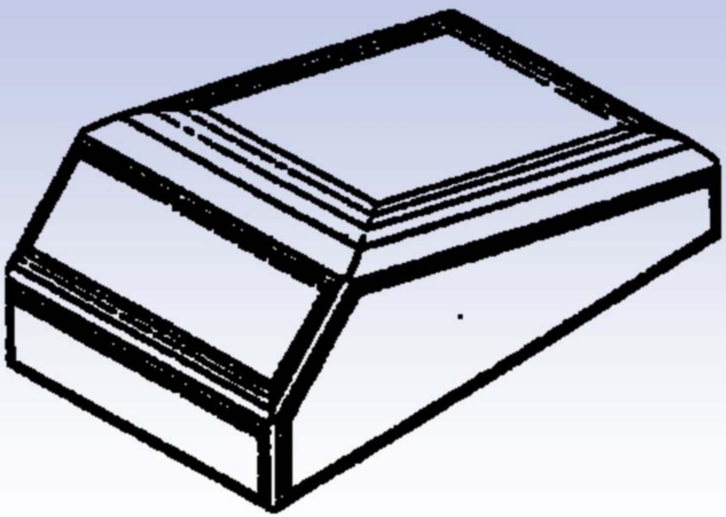
line and planar segments



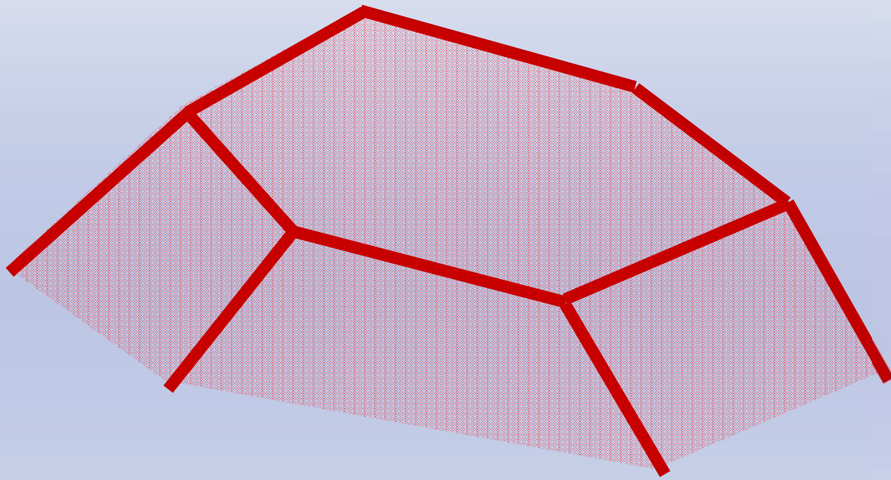


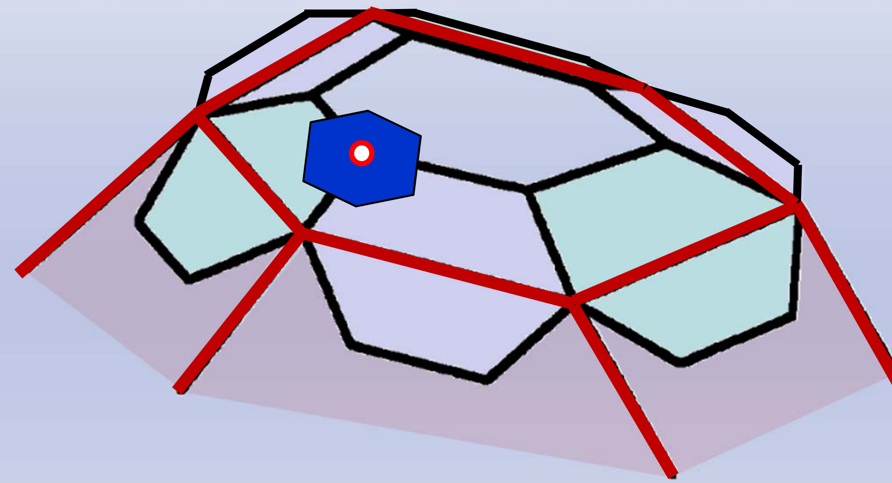




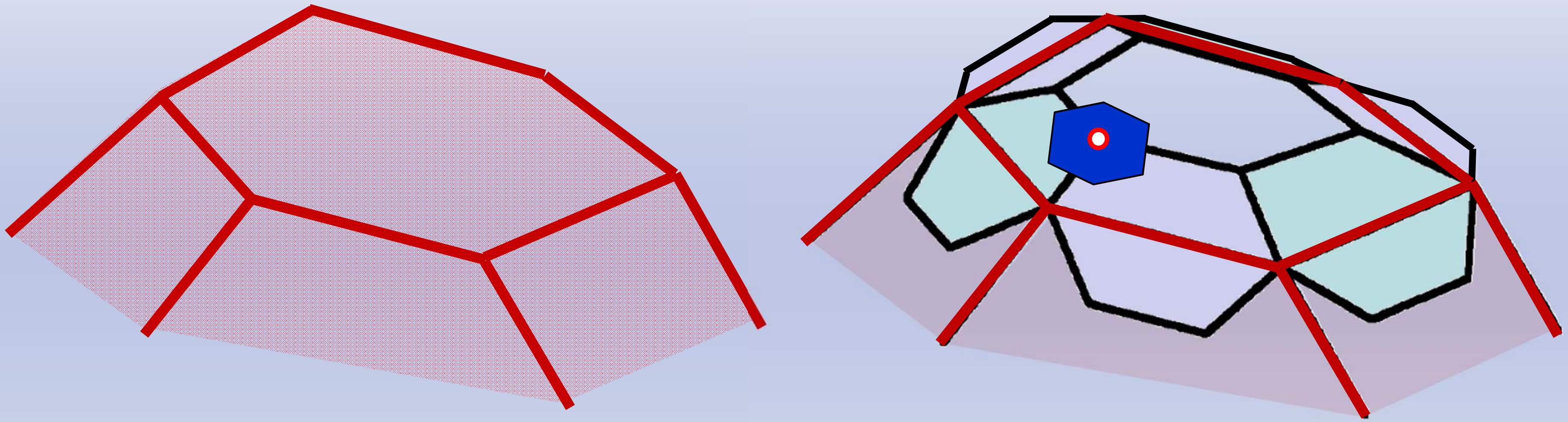


The contact elements of a honeycomb surface





The contact elements of a honeycomb surface



determine a corner cutting (cc) scheme
for the tangent polyhedra.

$\mathcal{H}$ is the only known cc scheme for surfaces

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cc is well-understood for curves

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little is known for surface cc

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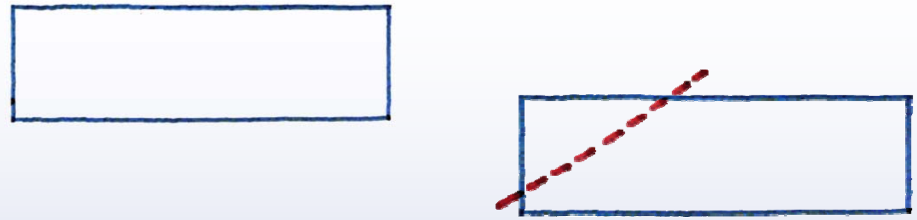
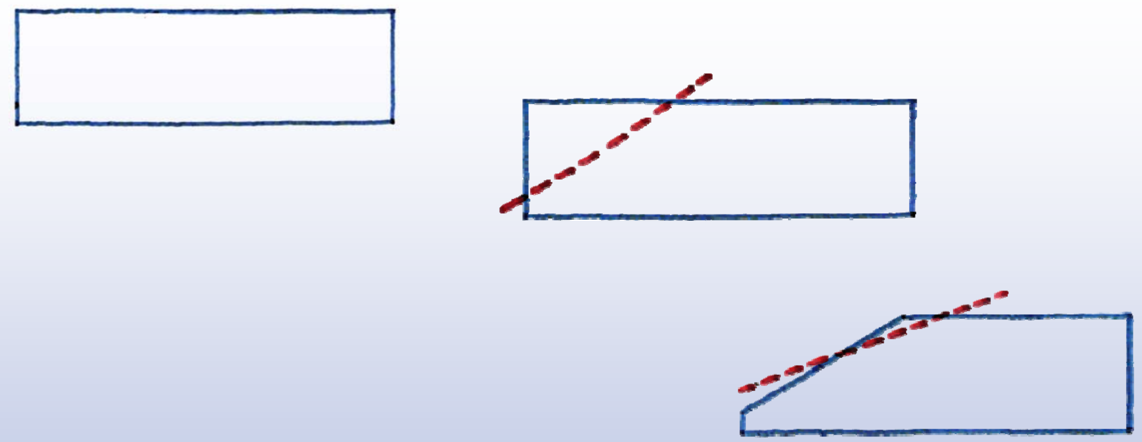
line segments



local corner cutting



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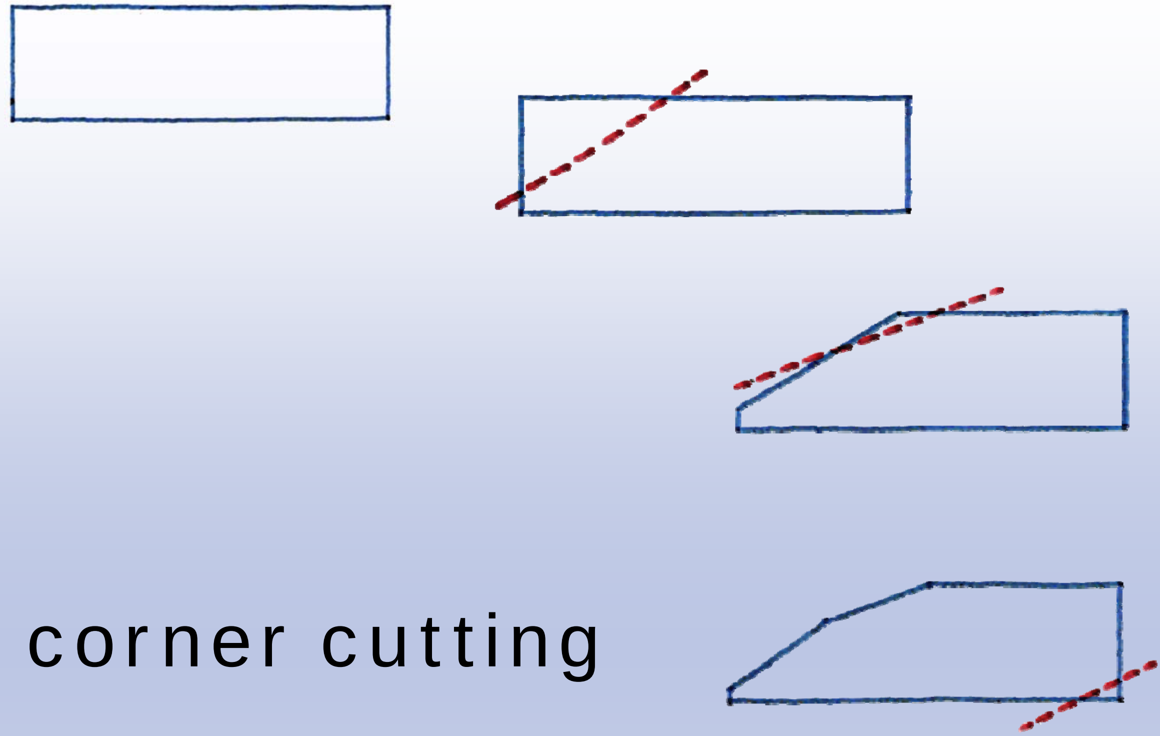

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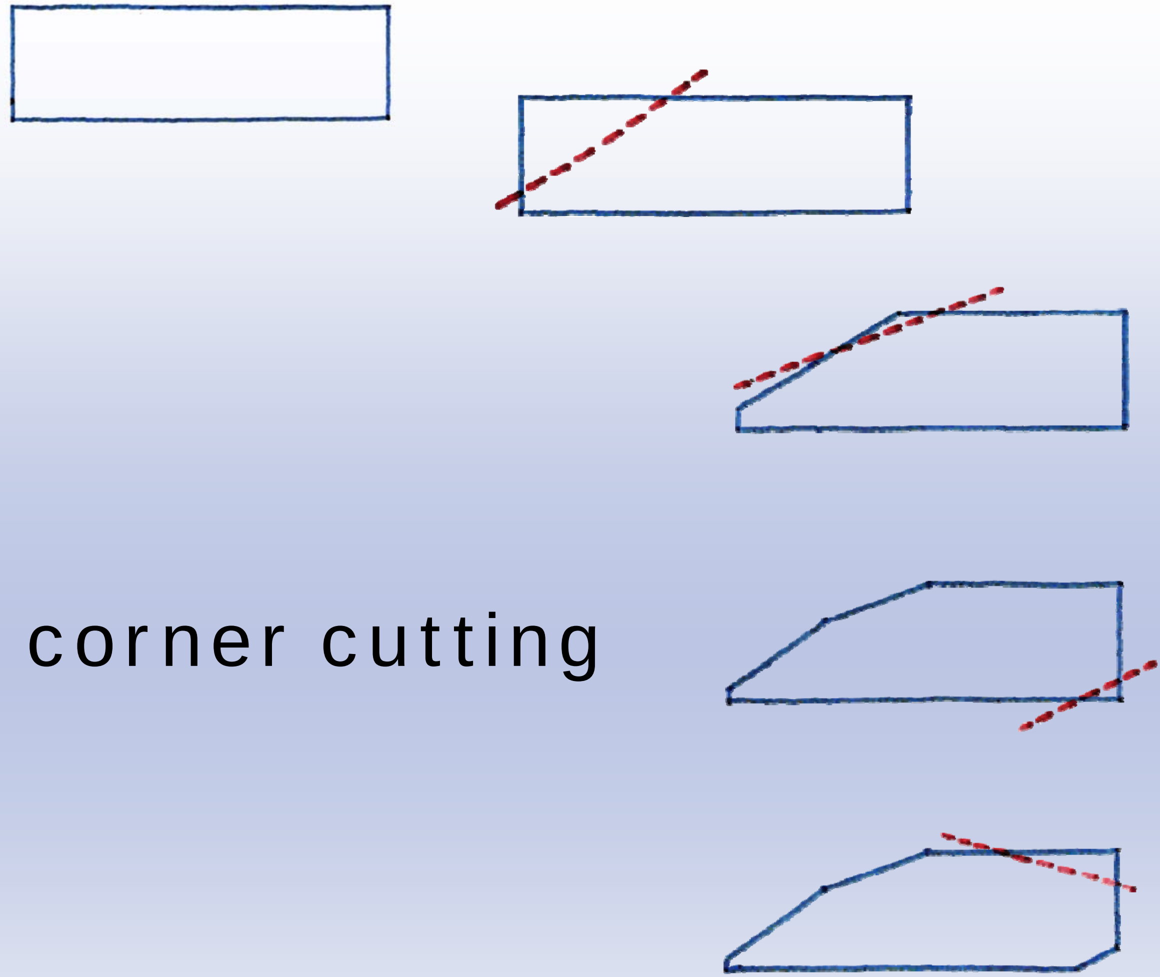

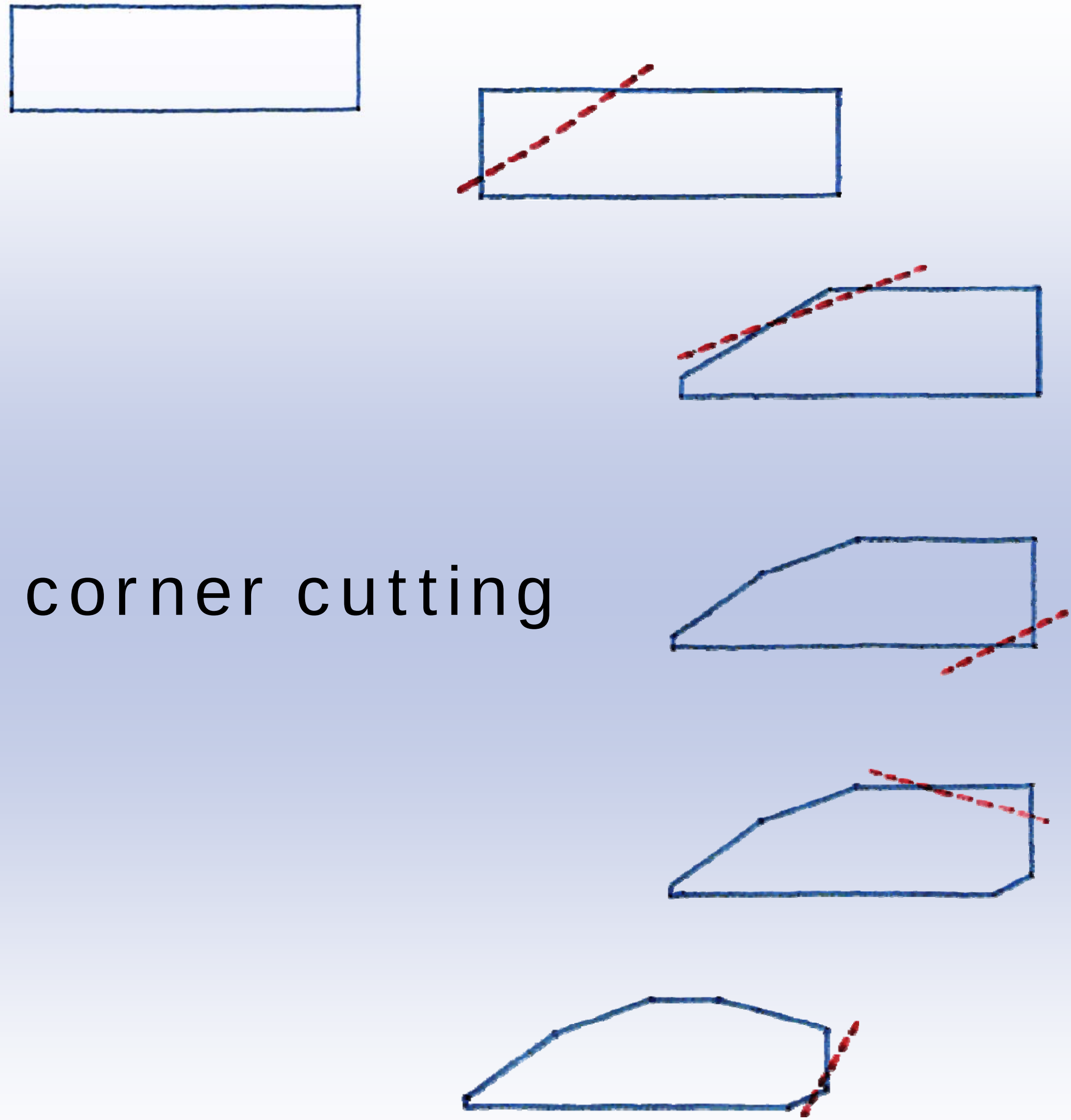

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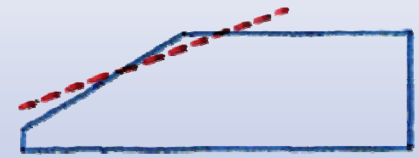

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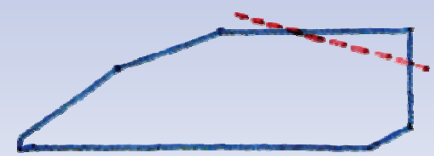
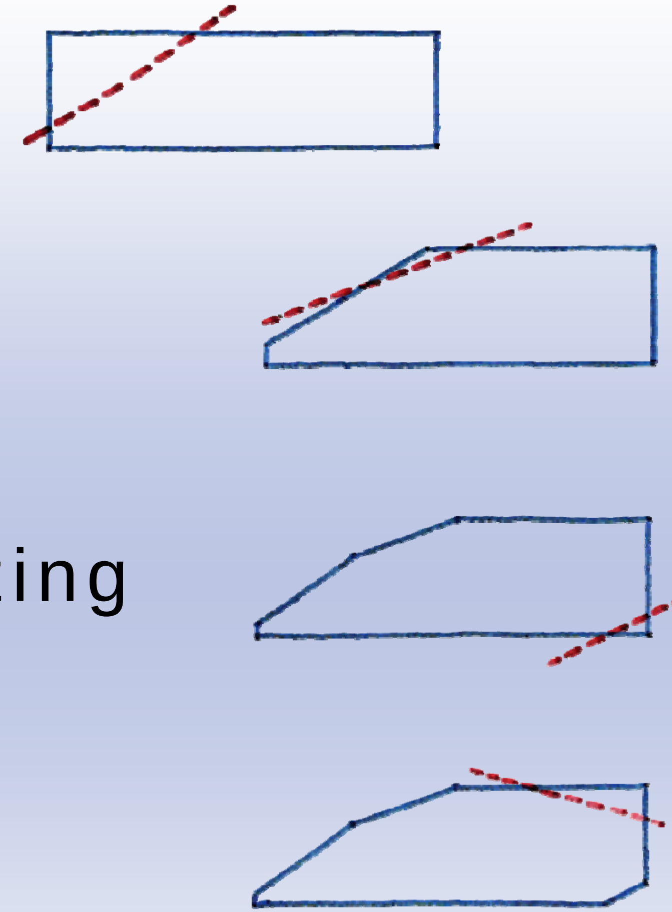
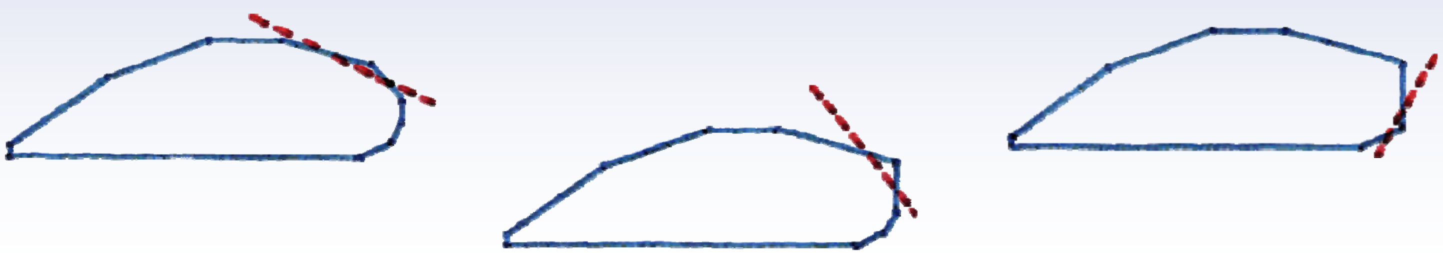
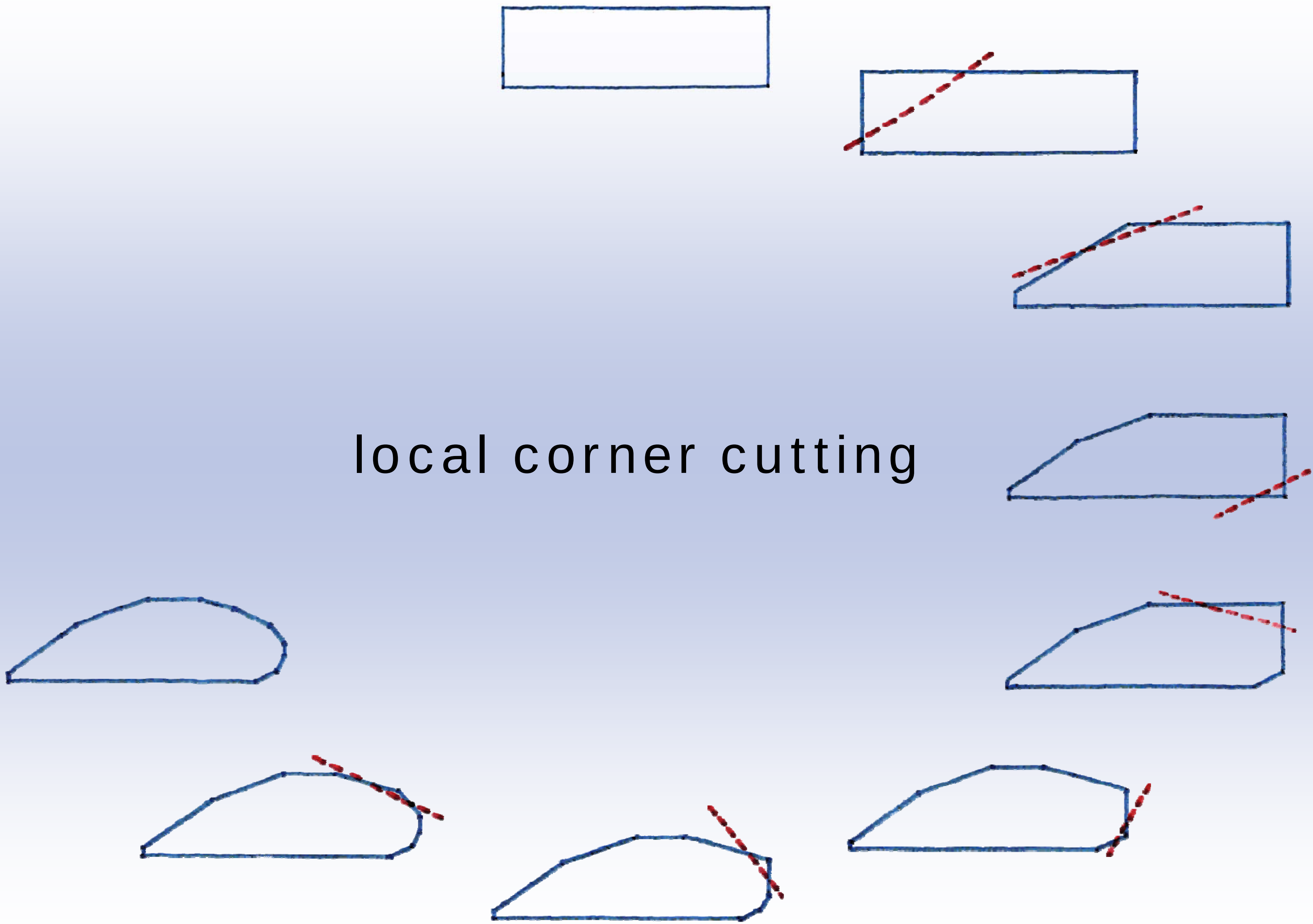
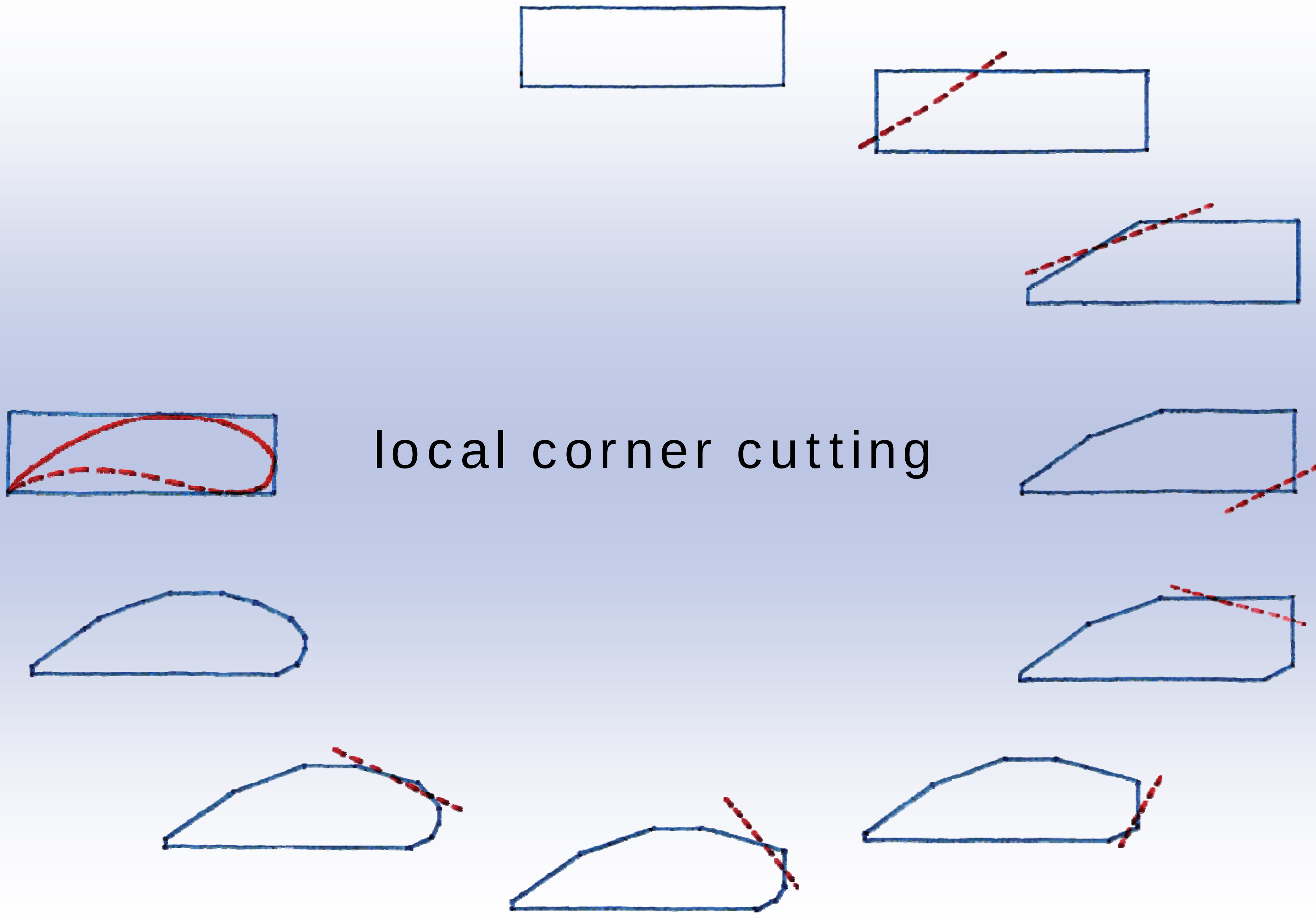




local corner cutting




local corner cutting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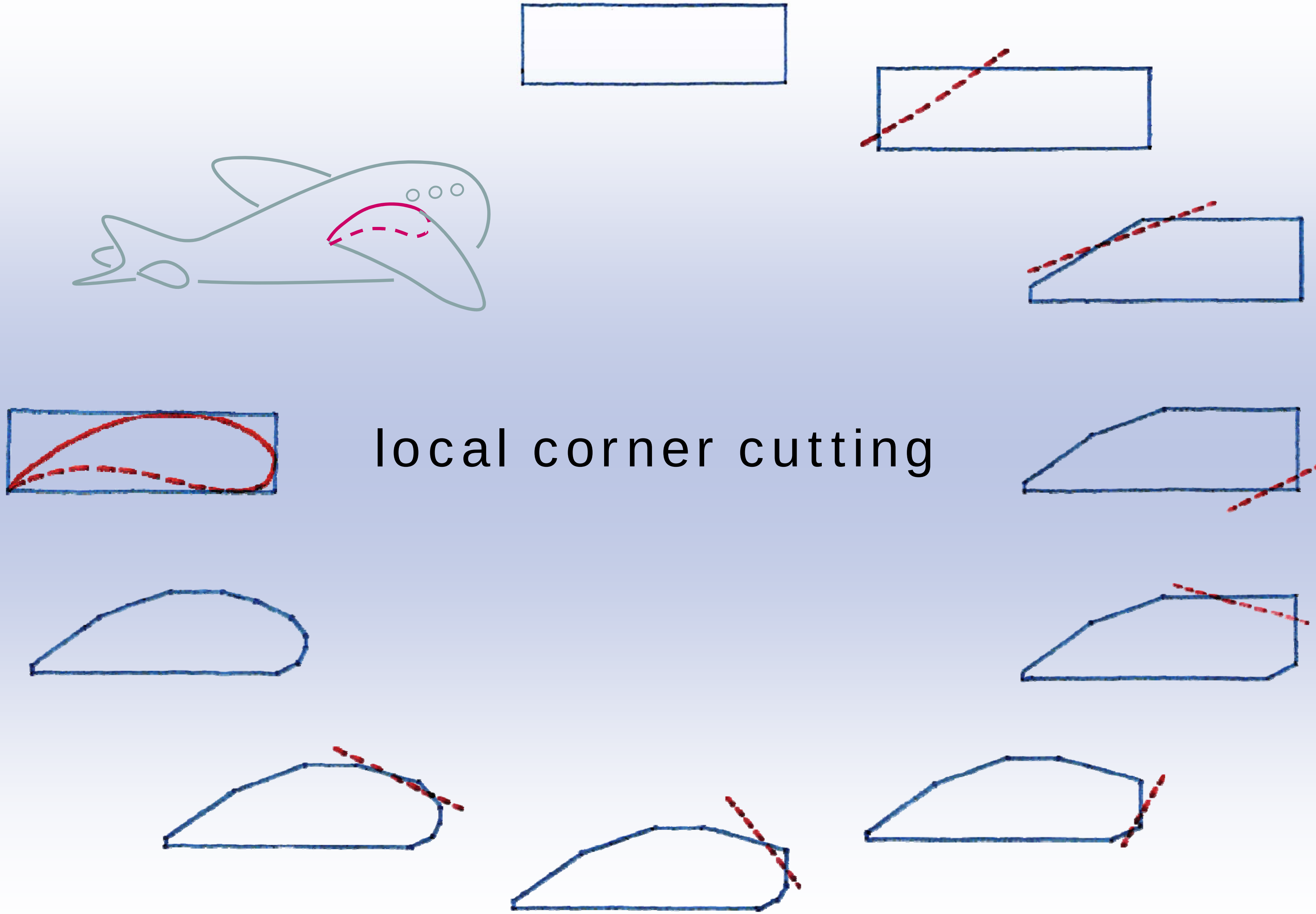

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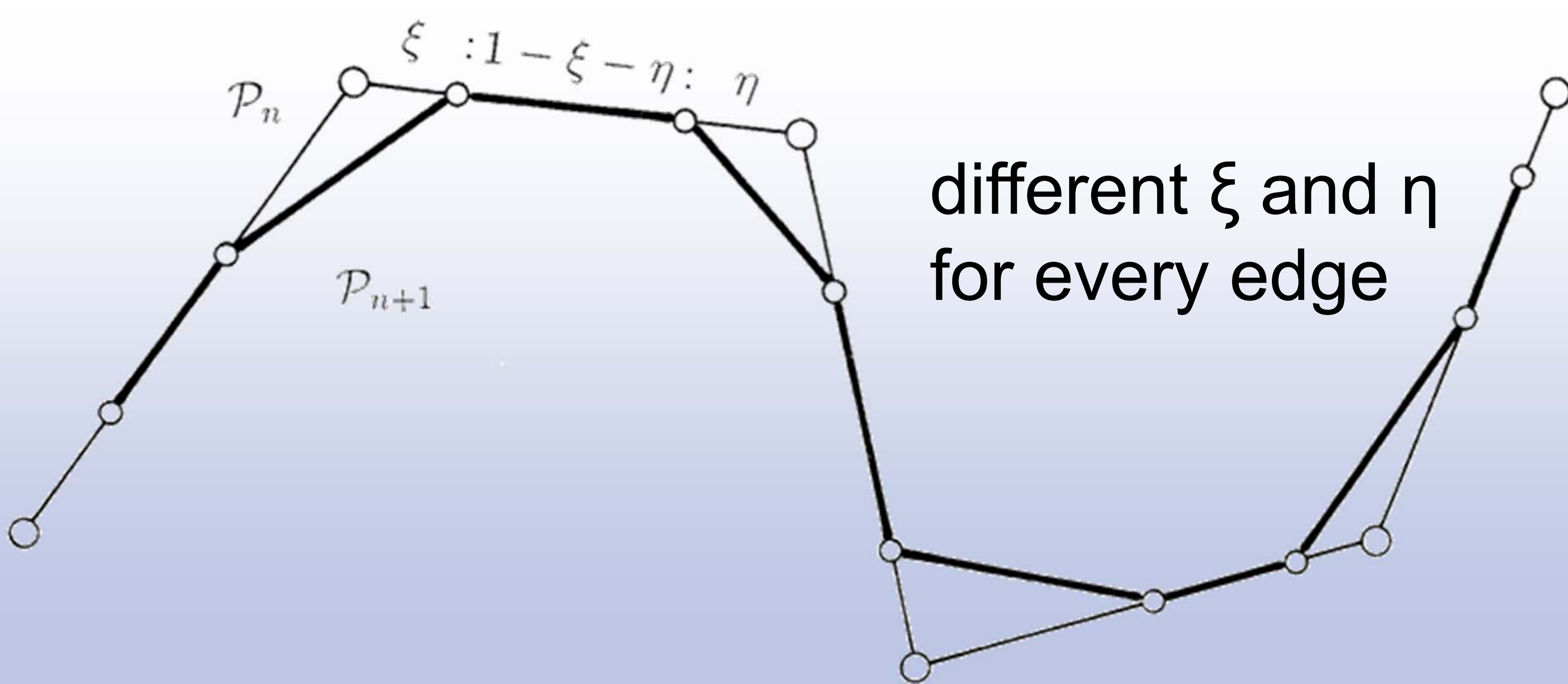
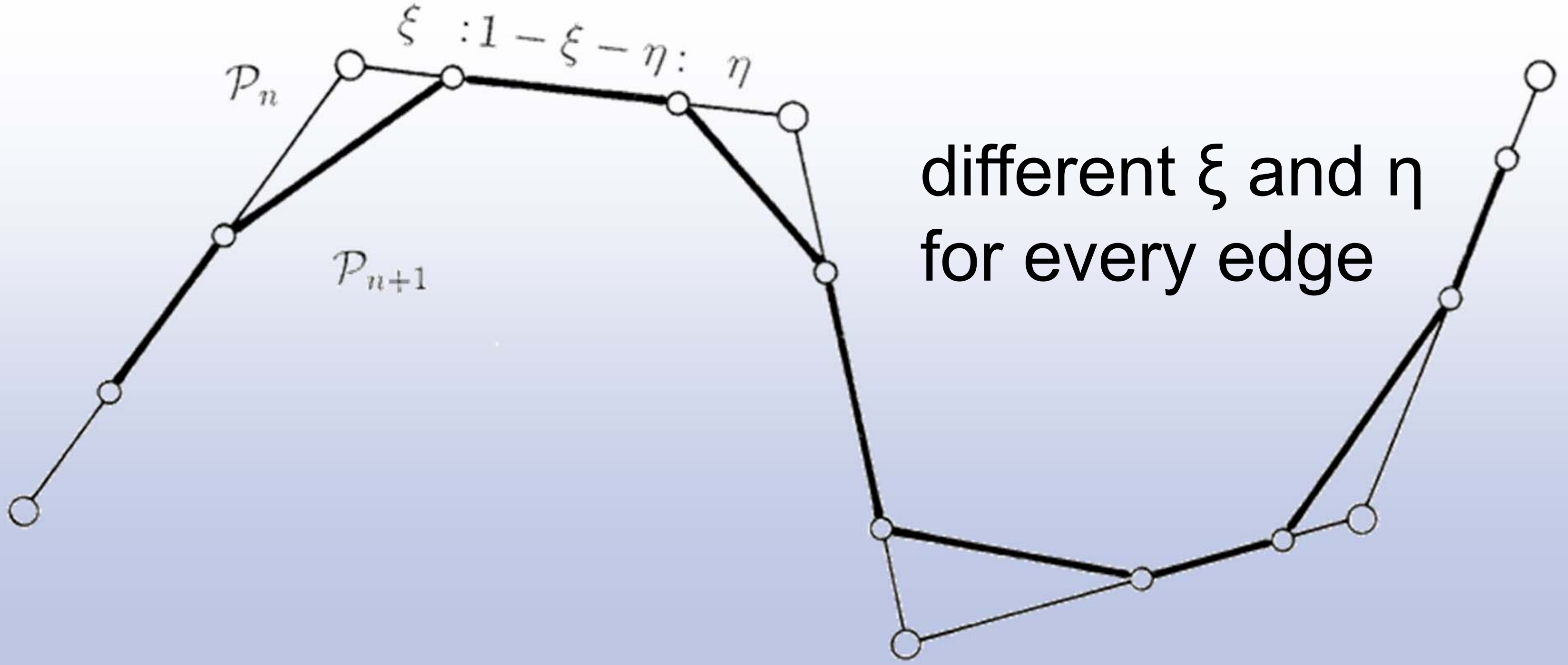


different ξ and η
for every edge



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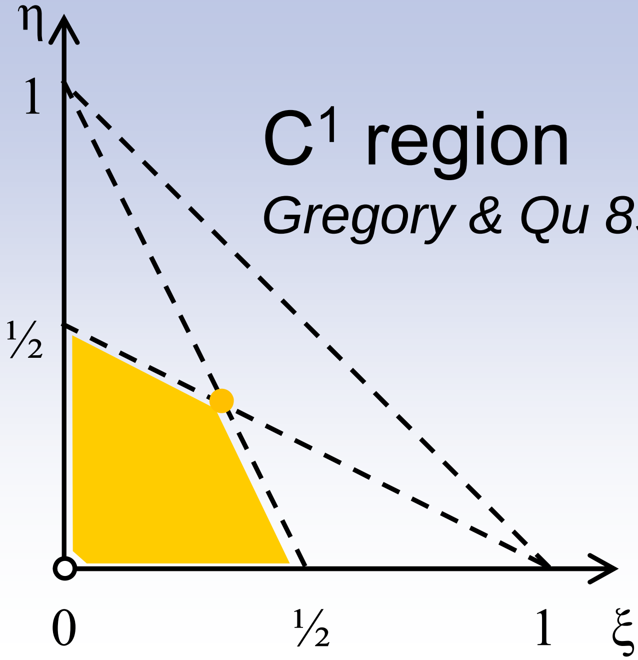
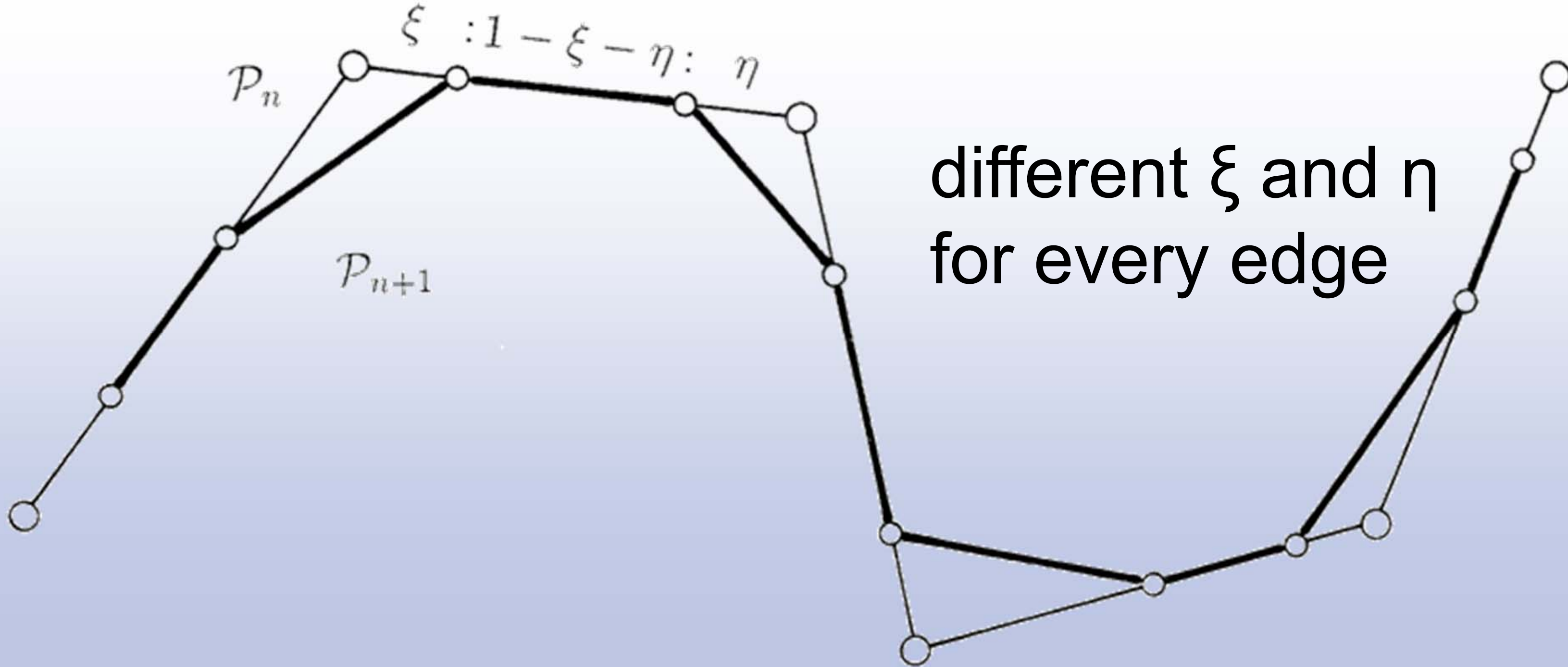


C¹ region

Gregory & Qu 89/96



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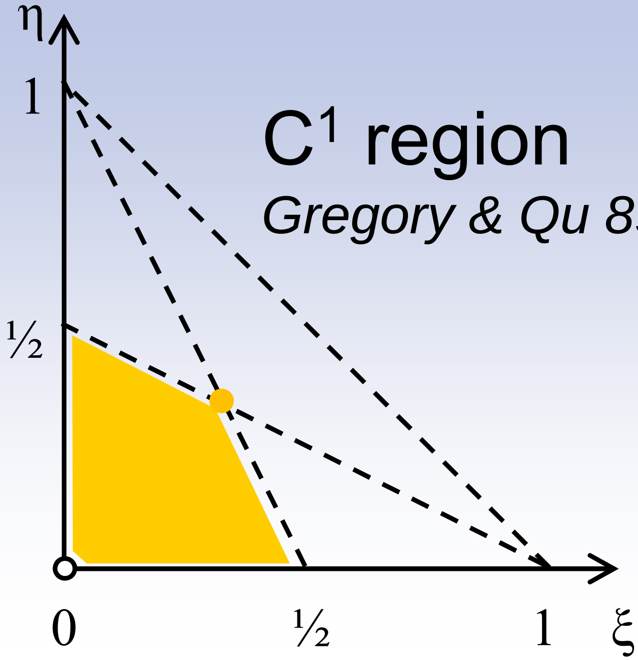
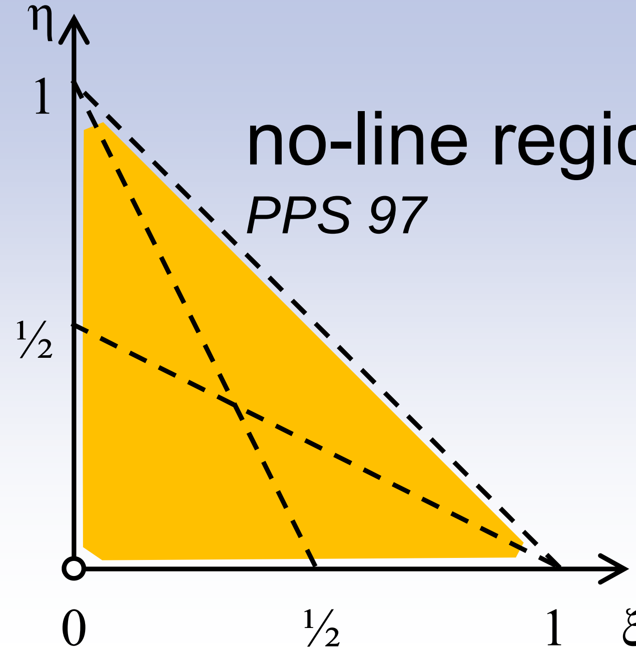


C^1 region

Gregory & Qu 89/96

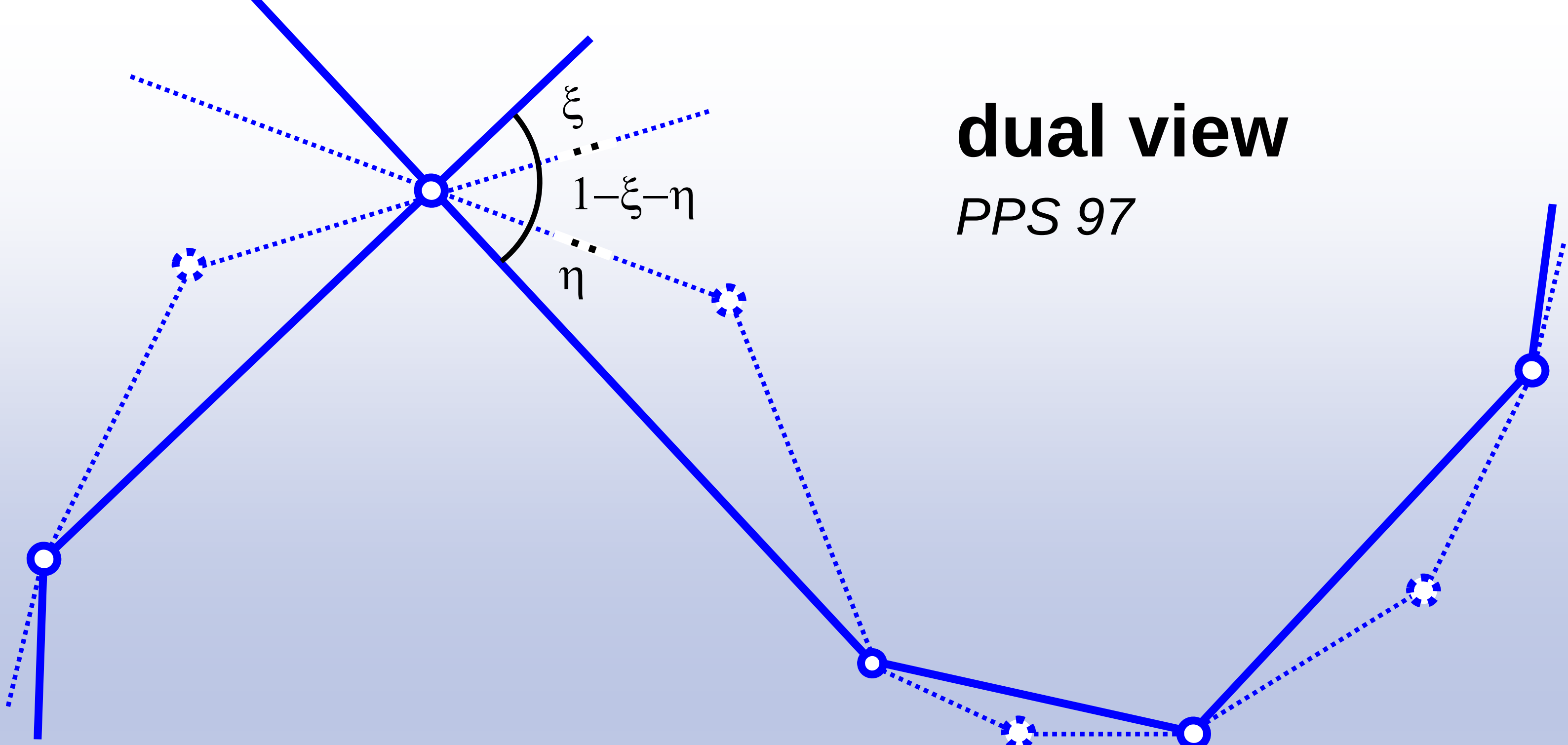


no-line region

PPS 97

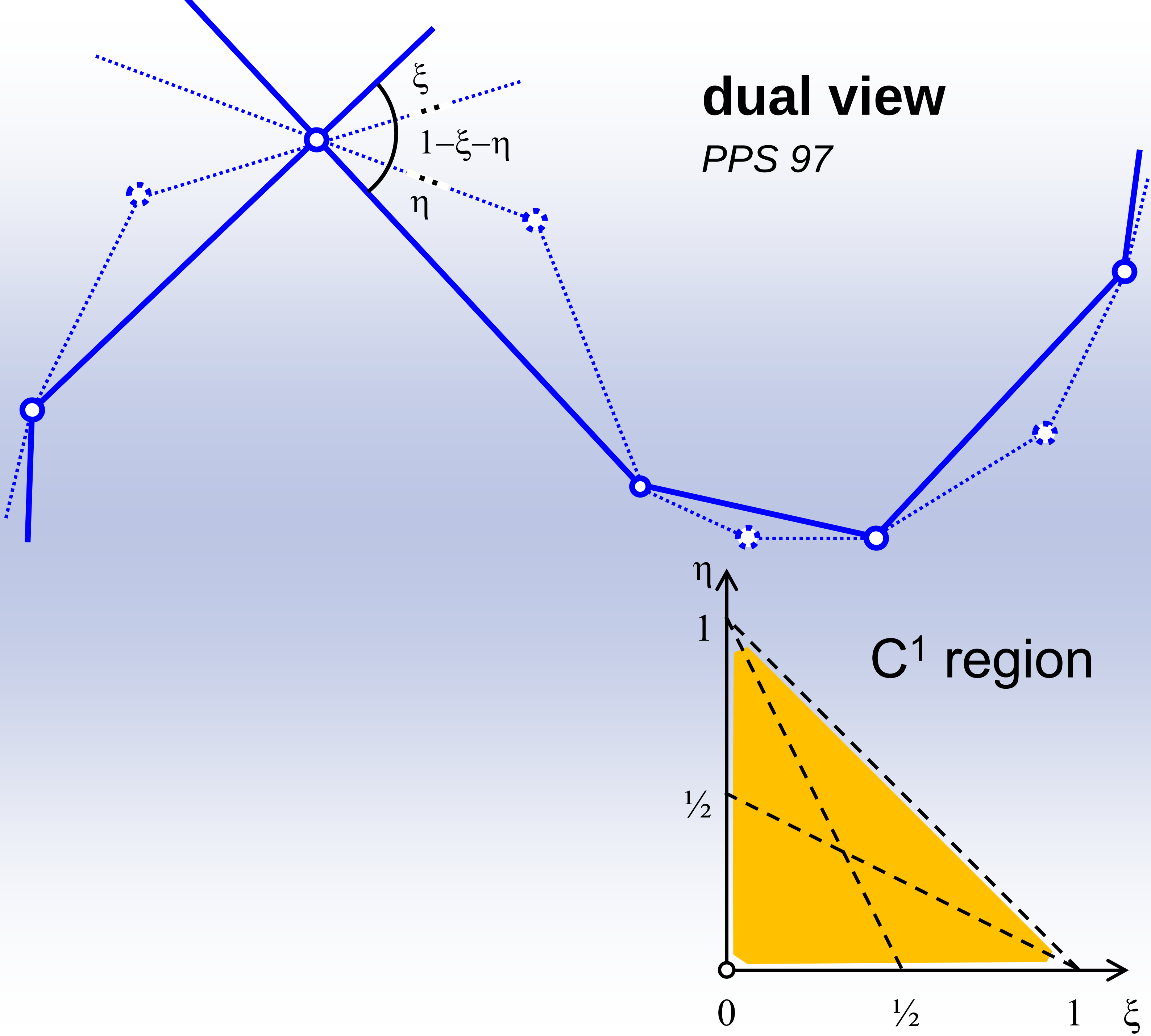
dual view

PPS 97



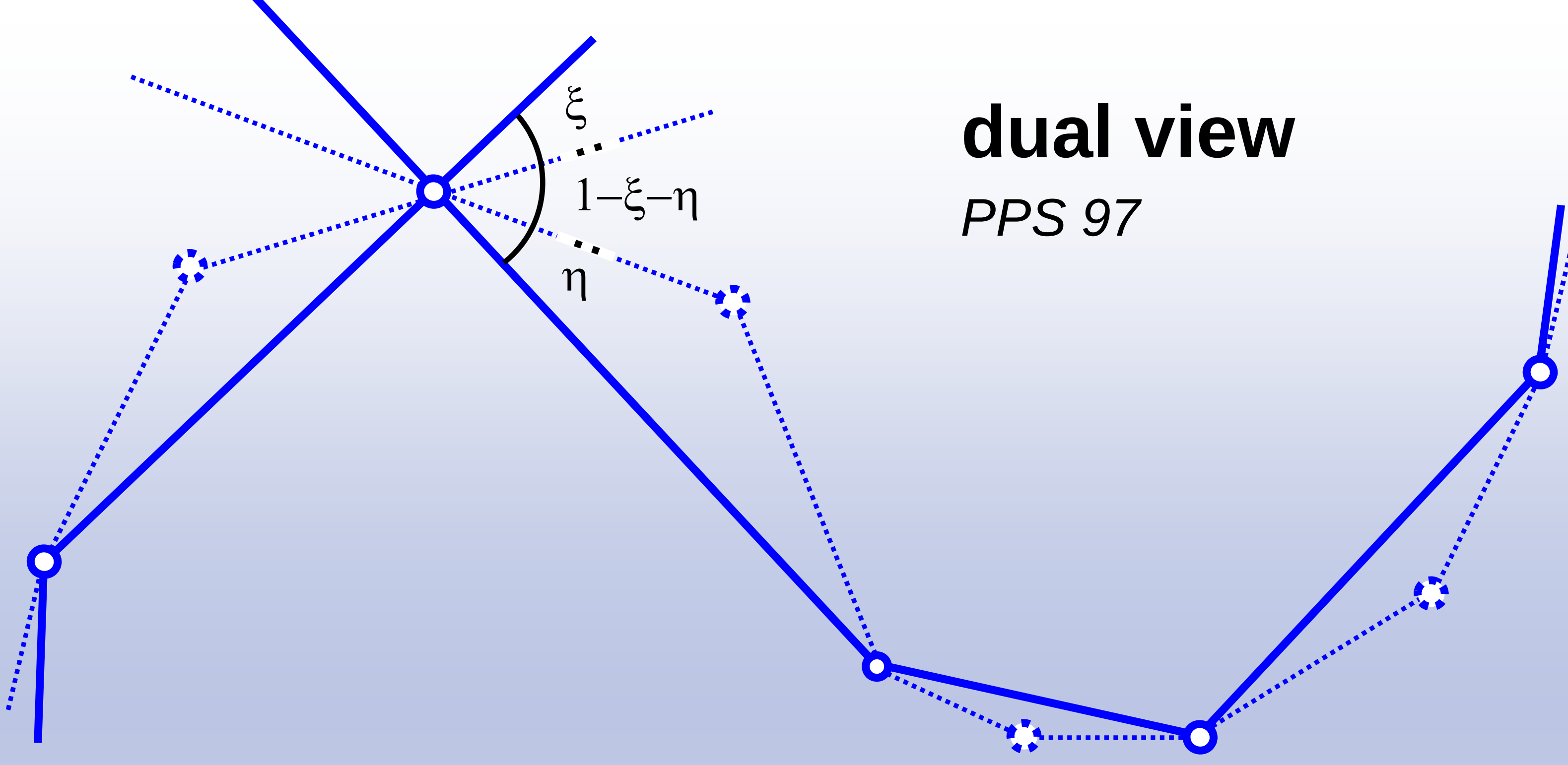
dual view

PPS 97

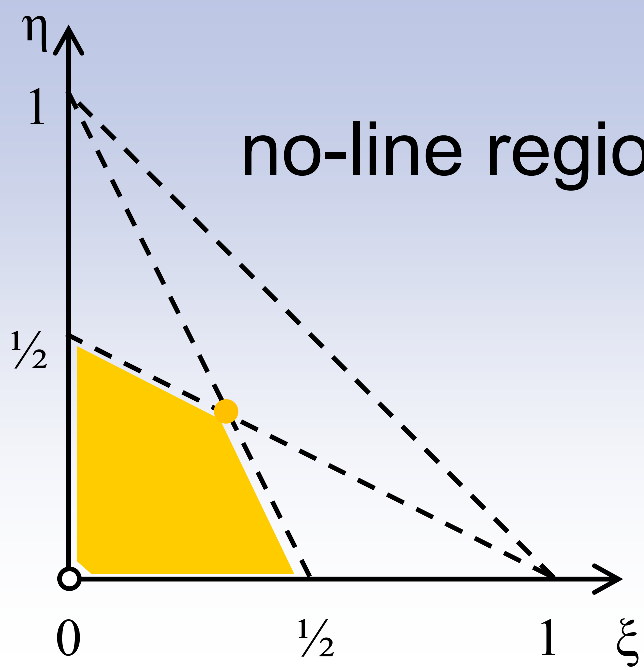


dual view

PPS 97



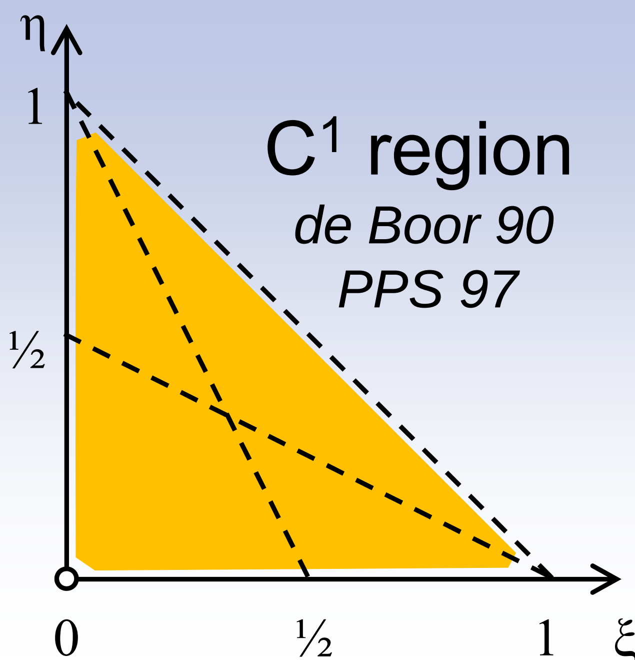
no-line region

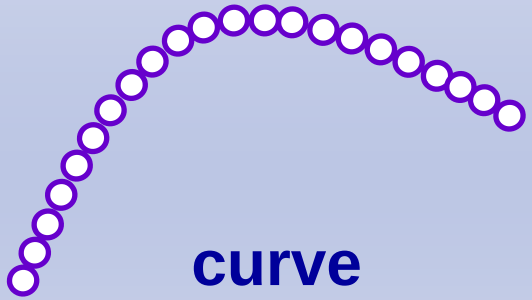


C^1 region

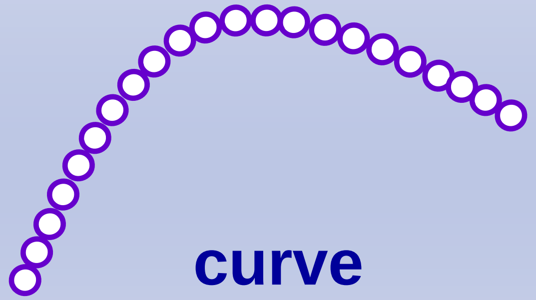
de Boor 90

PPS 97





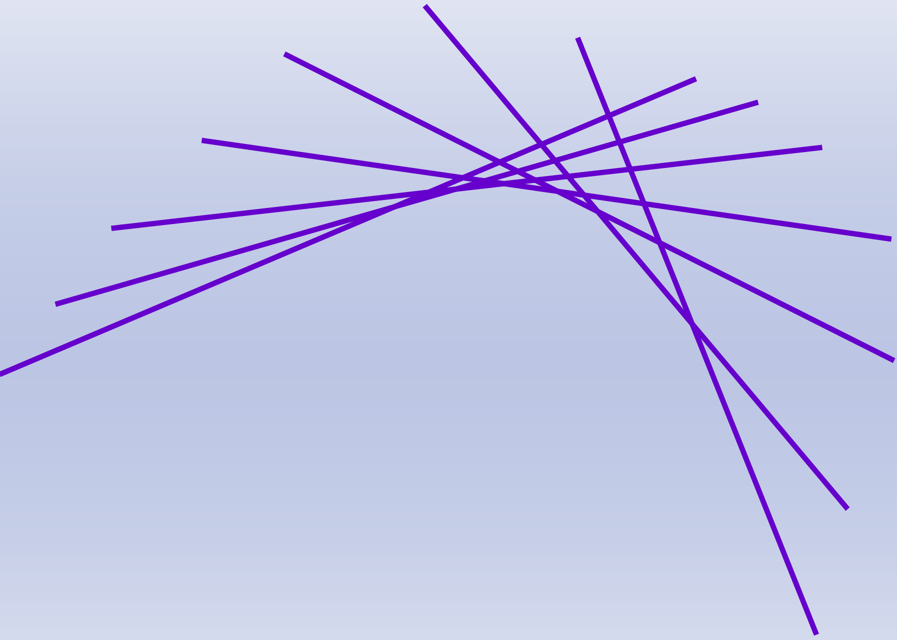
curve

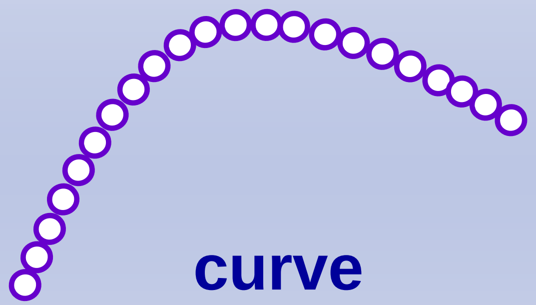


curve

$$\mathbf{u}^t\,\mathbf{x} = 1 = \mathbf{x}^t\,\mathbf{u}$$

dual lines





curve

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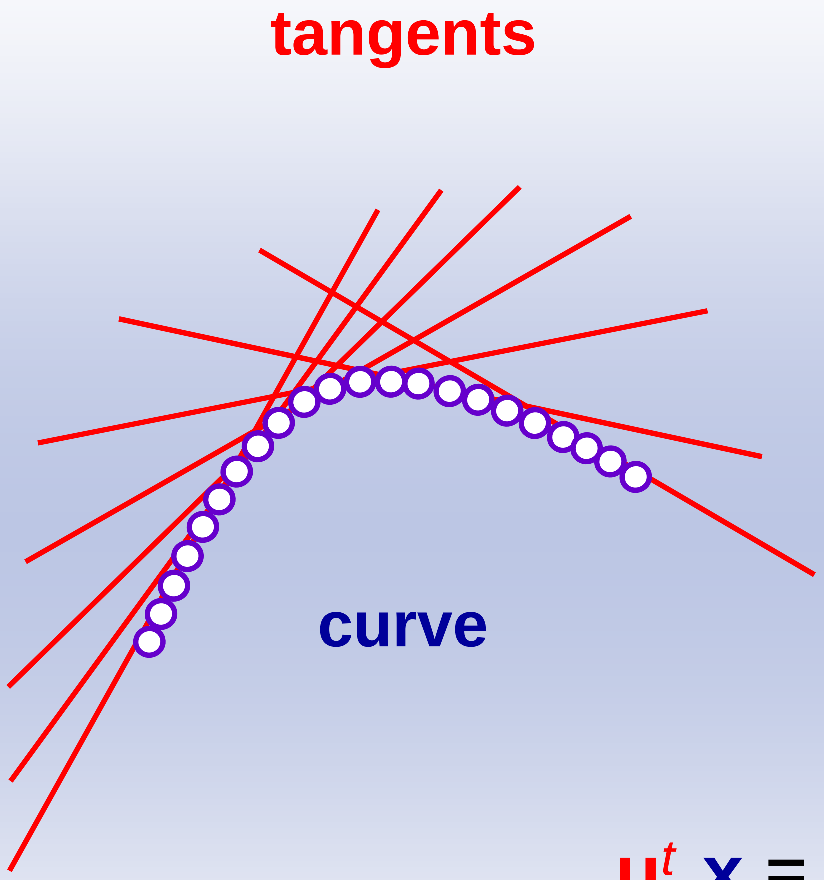
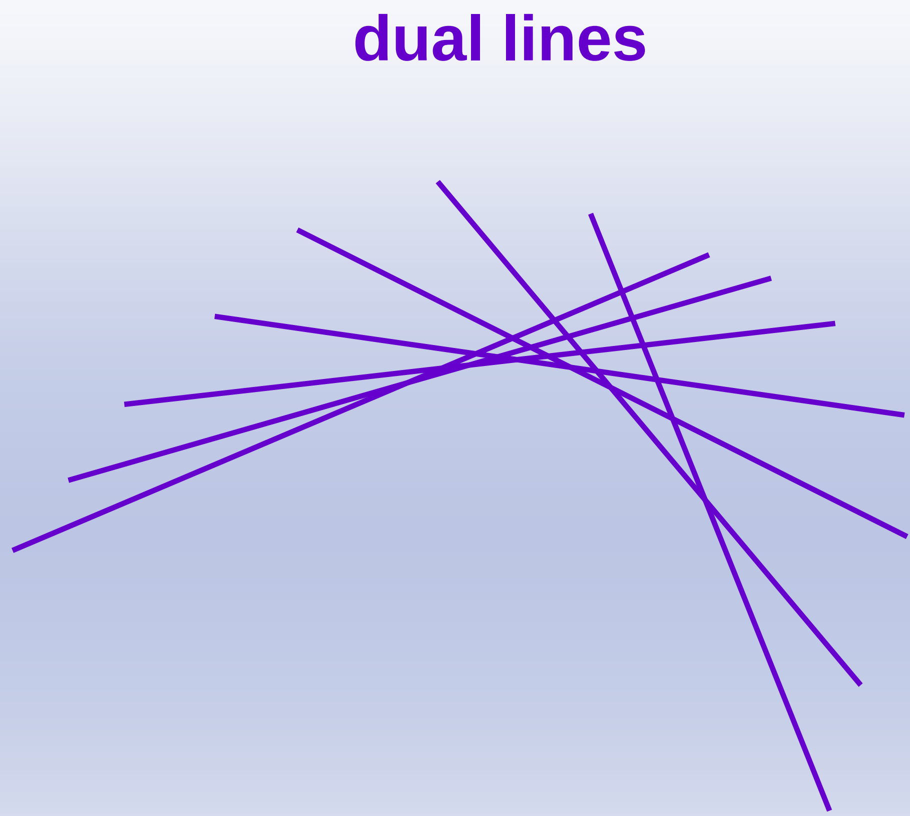
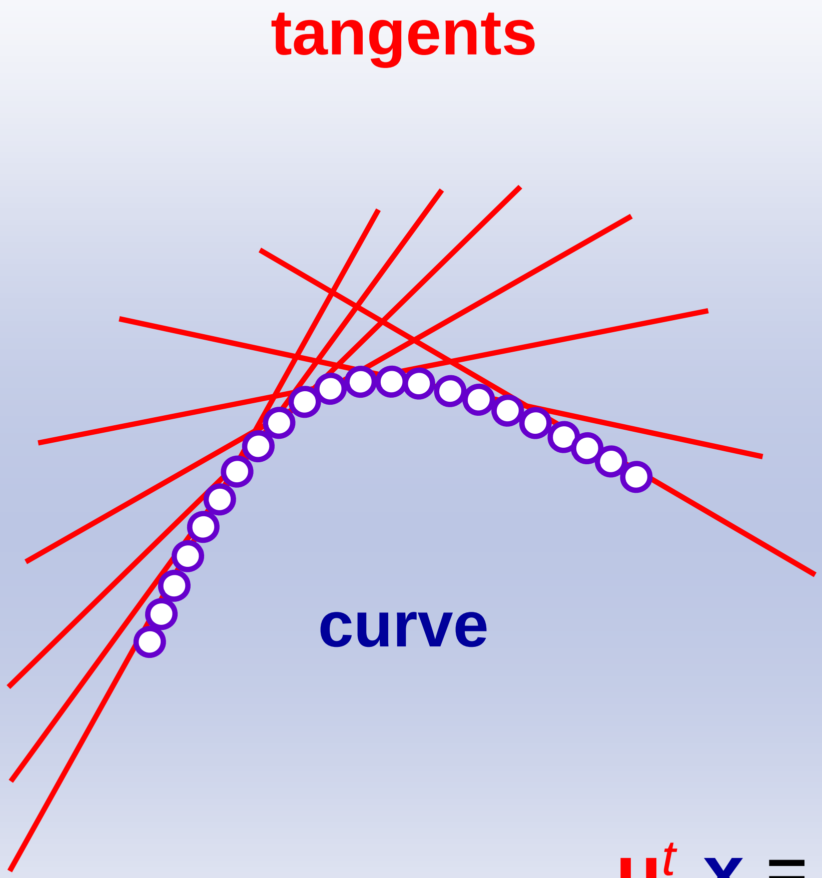
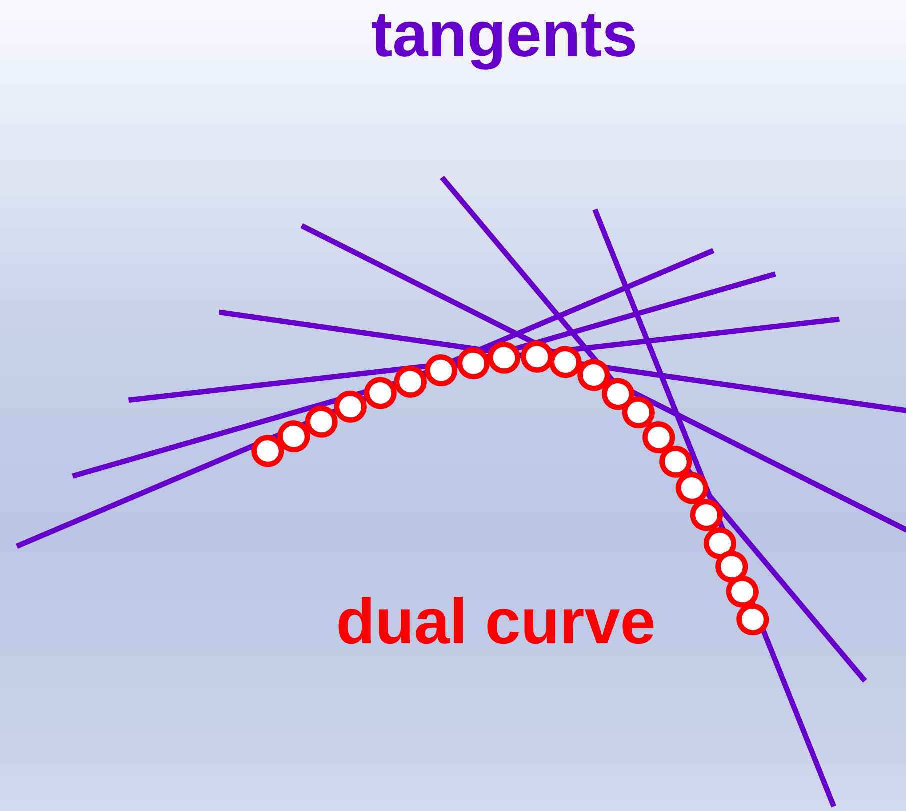




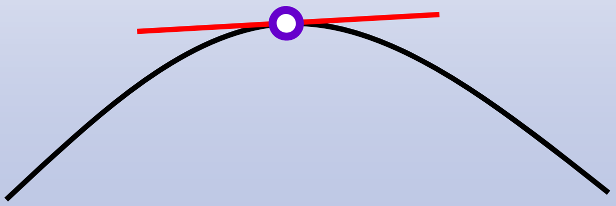
$$\mathbf{u}^t\, \mathbf{x} = 1 = \mathbf{x}^t\, \mathbf{u}$$



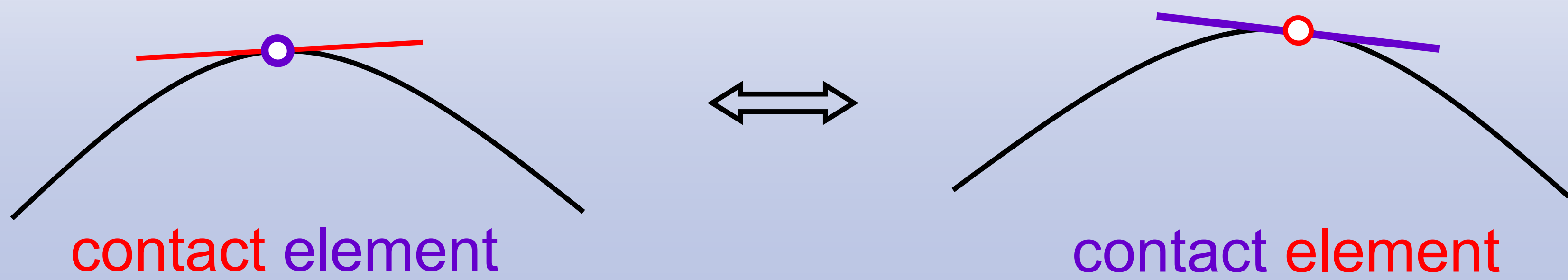


$$\mathbf{u}^{t}\ \mathbf{x} = 1 = \mathbf{x}^{t}\ \mathbf{u}$$

curve



curve **dual curve**



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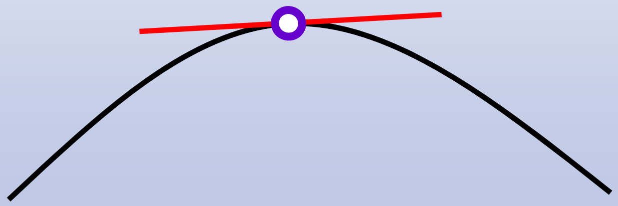
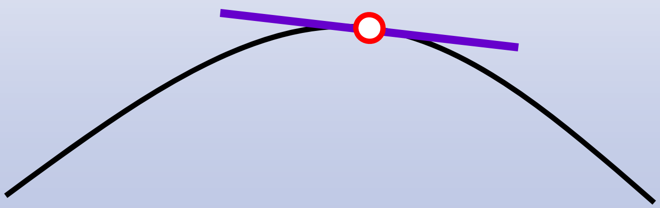
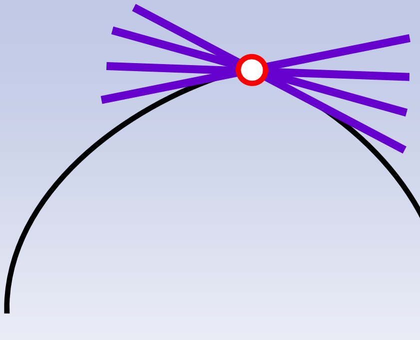




contact element contact element



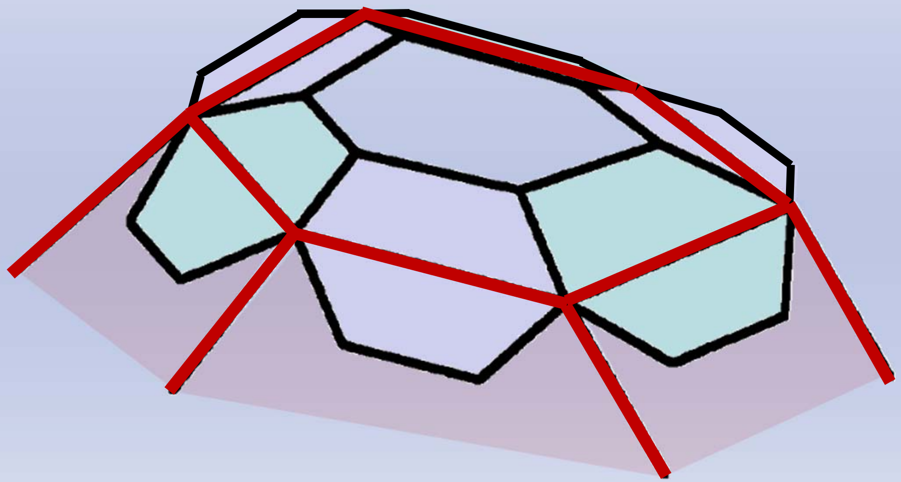
line segment corner point

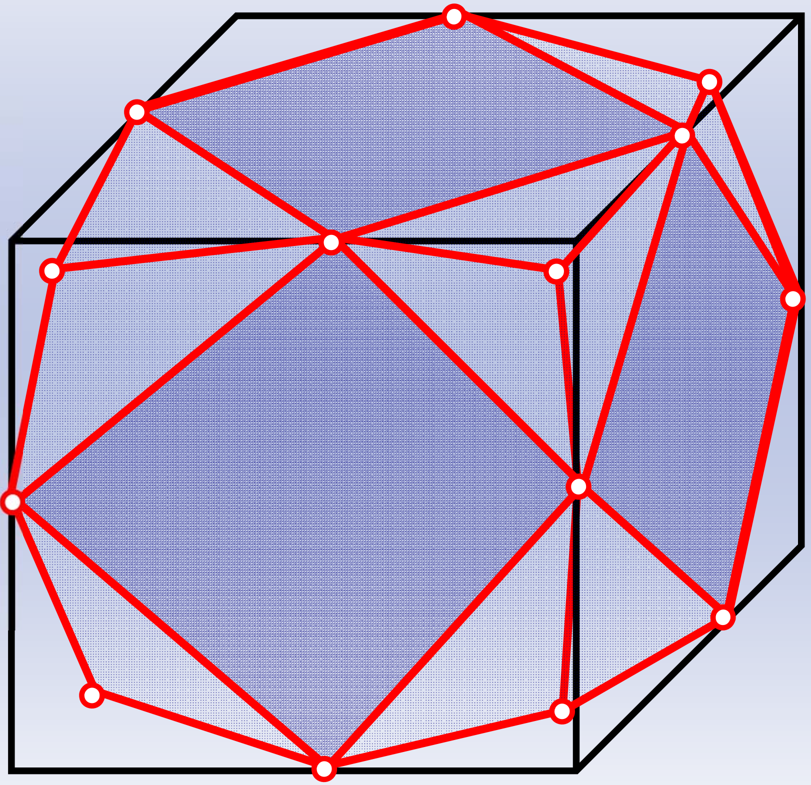
A convex curve is C^1

iff

the dual curve has no line segments.

The dual honeycomb scheme
(H^*)

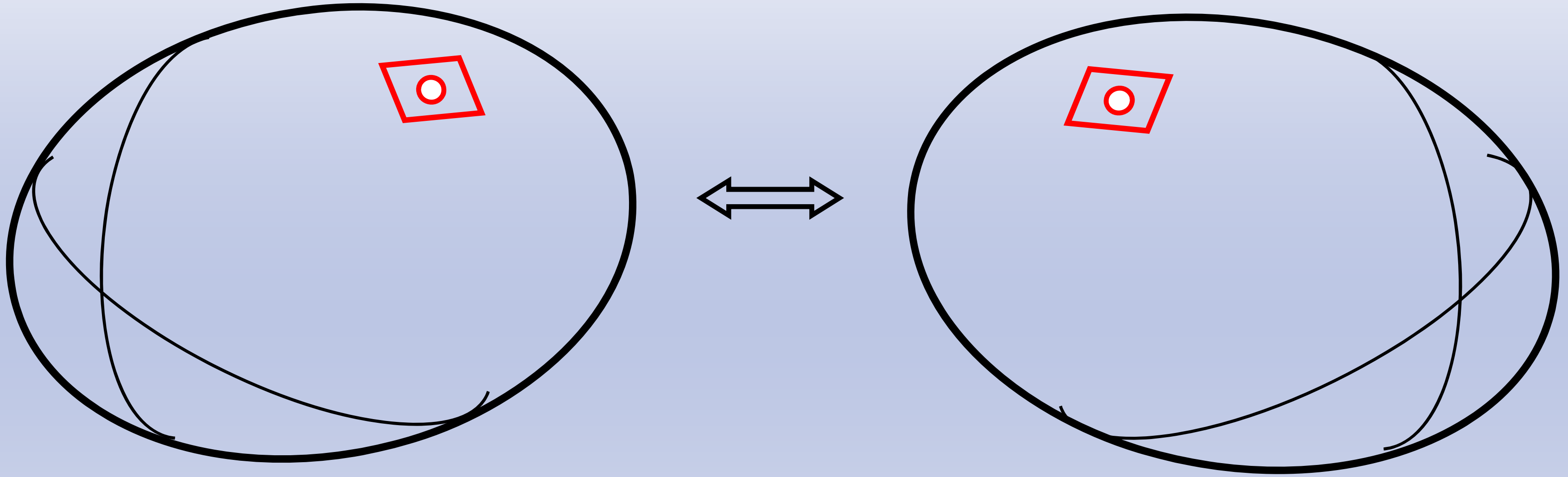




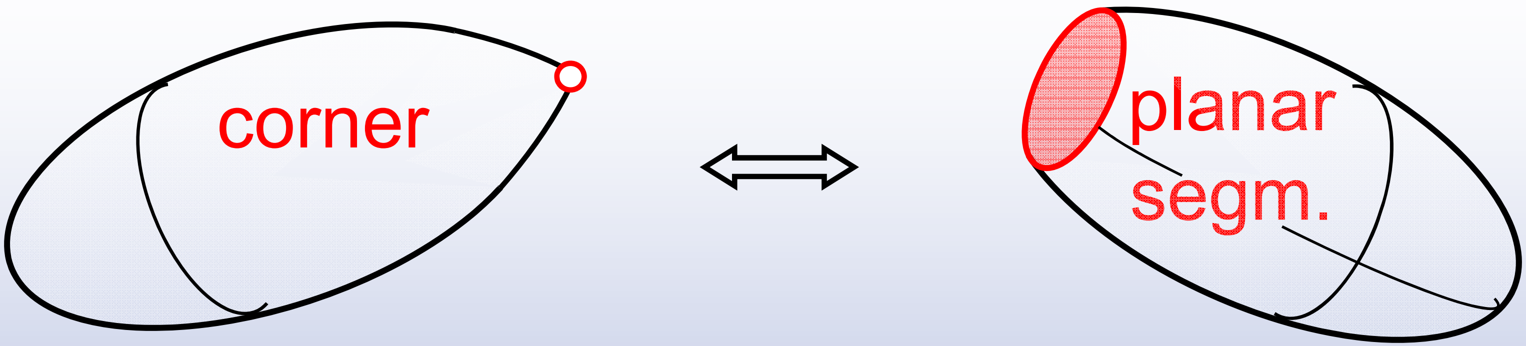
surface

dual surface

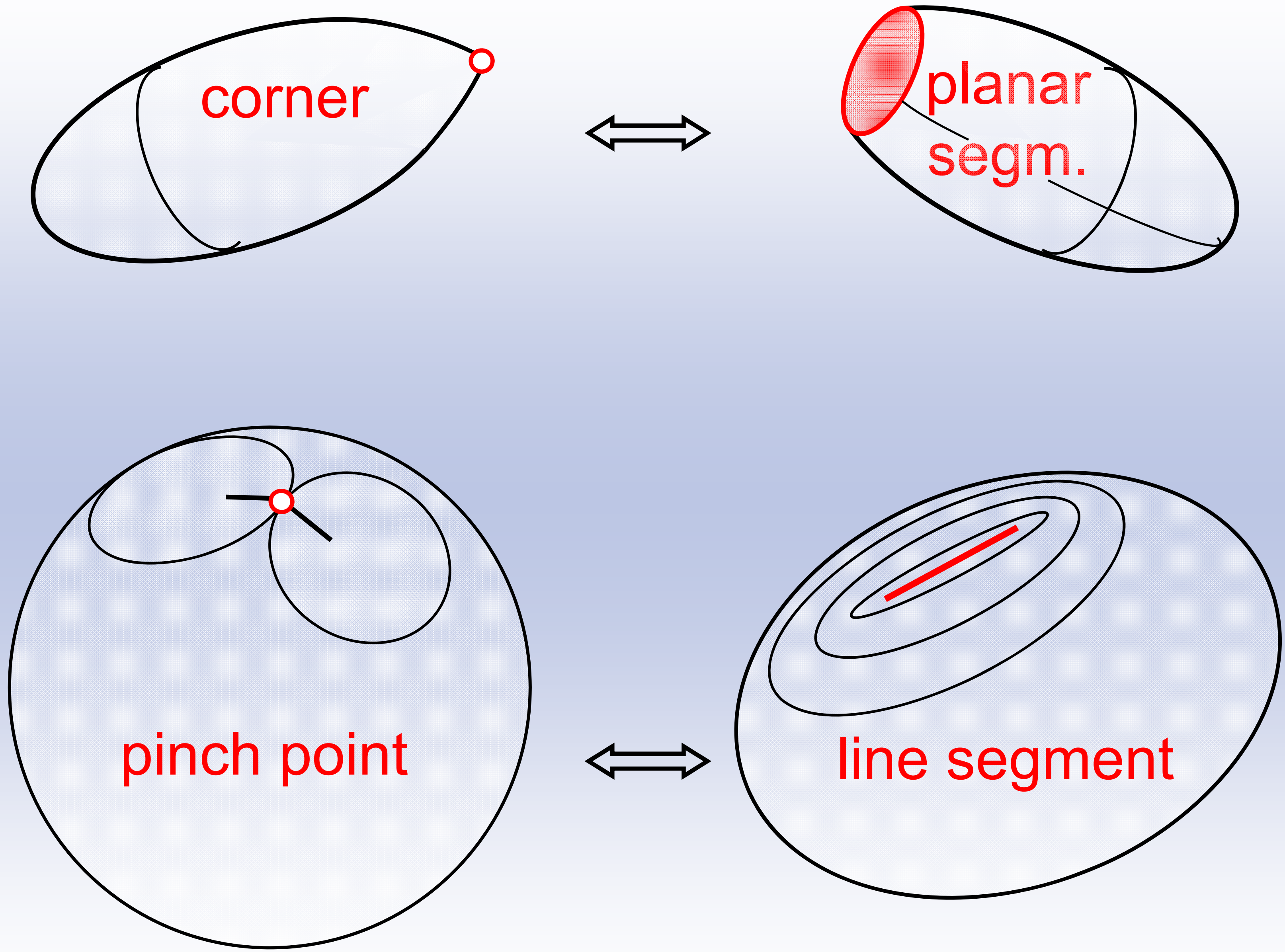
surface **dual surface**



contact element ⟺ contact element



corner
planar
segm.


corner
planar
segm.
pinch point
line segment

A convex surface is C^1

iff

the dual surface has no line segments.

A convex surface is C^1

iff

the dual surface has no line segments.

$\mathcal{H}^*$ generates no line segments,

but corners and pinch points.

For C^1 surfaces w/o line segm.

combine $\mathcal{H}$ and $\mathcal{H}^*$

to

Mixed **P**rimal **D**ual honeycomb schemes

($\mathcal{M}_{pd}$)

1st attempt

alternate

$\boldsymbol{\mathcal{H}}$ on the secant poyhedra with

$\boldsymbol{\mathcal{H}}^*$ on the tangent polyhedra

1st attempt

alternate

$\mathcal{H}$ on the secant poyhedra with

$\mathcal{H}^*$ on the tangent polyhedra

problems

- unknown tangent planes

1st attempt

alternate

$\mathcal{H}$ on the secant poyhedra with

$\mathcal{H}^*$ on the tangent polyhedra

problems

- unknown tangent planes
- topology of polyhedra unpredictable

1st attempt

alternate

$\mathcal{H}$ on the secant poyhedra with

$\mathcal{H}^*$ on the tangent polyhedra

problems

- unknown tangent planes
- topology of polyhedra unpredictable
- $\mathcal{H}^*$ may increase angles
- $\mathcal{H}$ may increase edge lengths

2nd attempt

alternate $\boldsymbol{\mathcal{H}}$ and $\boldsymbol{\mathcal{H}}^*$

both on the secant poyhedra

$$\boldsymbol{\mathcal{M}}_{pd} := \boldsymbol{\mathcal{H}}^{p}\,(\boldsymbol{\mathcal{H}}^*)^{d}$$

2^{nd} attempt

alternate $\boldsymbol{H}$ and $\boldsymbol{H}^*$

both on the secant poyhedra

$$\mathcal{M}_{pd} := \boldsymbol{H}^{\,p}\,(\boldsymbol{H}^*)^d$$

Then

$\mathcal{M}^*_{pd}$ and $\mathcal{M}_{dp}$

generate the same set of surfaces

2^{nd} attempt

alternate $\boldsymbol{H}$ and $\boldsymbol{H}^*$

both on the secant poyhedra

$$\boldsymbol{M}_{pd} := \boldsymbol{H}^{p} (\boldsymbol{H}^*)^{d}$$

Then

$\boldsymbol{M}^*_{pd}$ and $\boldsymbol{M}_{dp}$

generate the same set of surfaces

Thus,

surfaces have no lines iff they are C^1

Theorem

For $\quad p > \left\lfloor \frac{\text{max. vertex valence}}{2} \right\rfloor$

and $\quad d > \left\lfloor \frac{\text{max. face valence}}{2} \right\rfloor$,

$\boldsymbol{\mathcal{M}}_{pd}$ generates

convex

C^1 surfaces

w/o line segm.

Remarks

Remarks

- polyhedra generated are not nested

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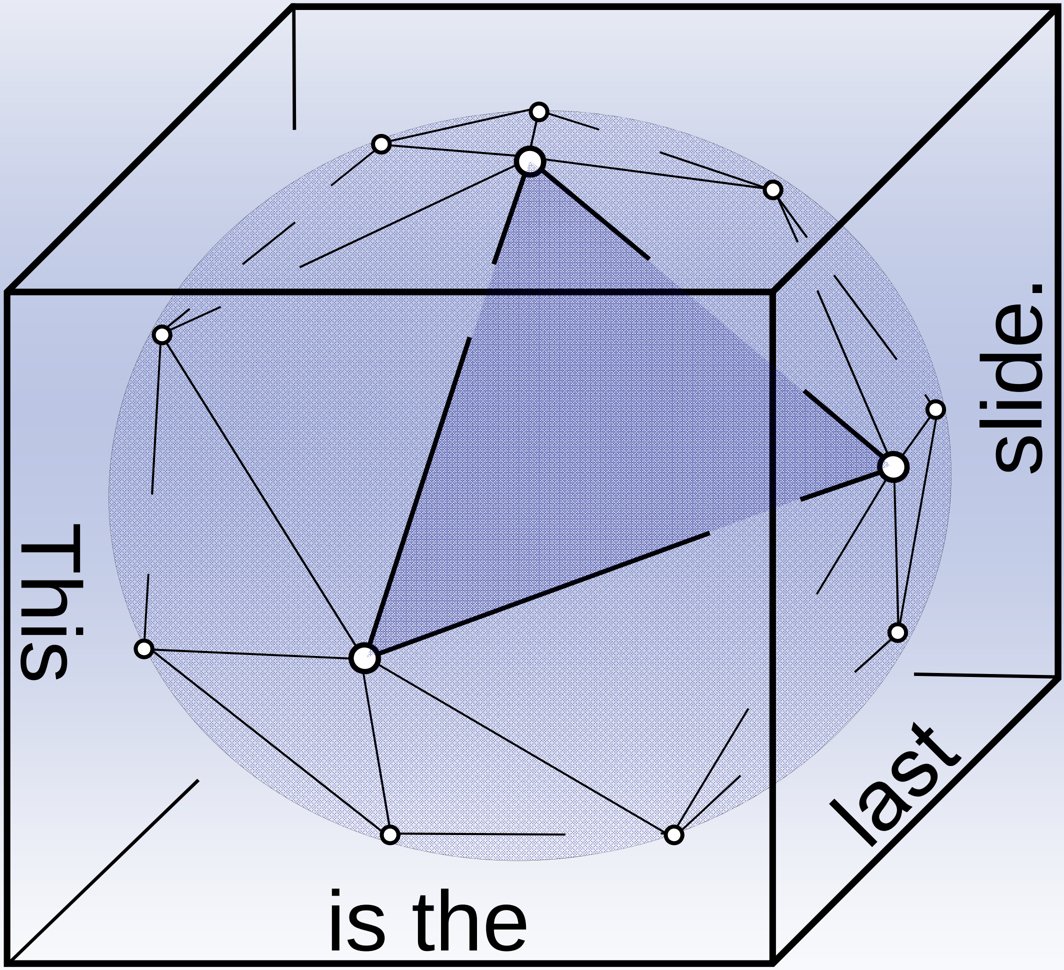
- polyhedra generated are not nested
- angles may increase
- but total face angles shrink under $\boldsymbol{H}^p$

Remarks

- polyhedra generated are not nested
- angles may increase
- but total face angles shrink under $\mathcal{H}^p$
- polyhedral parts that define a neighborhood?


This
is the
last
slide.