3rd IM-Workshop on

Applied Approximation, Signals and Images

Bernried (Germany)
February 19–23, 2018

ABSTRACTS

Organizers:
Costanza Conti
Mariantonia Cotronei
Nira Dyn
Brigitte Forster
Tomas Sauer

PROGRAM

Monday, February 19	
09:30–09:40	*Welcome & Opening*
09:40–10:30	Getting to know each other: Mathematical speed dating I
10:30–11:00	**Coffee break**
11:00–12:00	Getting to know each other: Mathematical speed dating II
15:00–15:30	**Coffee & cake**
15:30–16:15	Tom Lyche: *Tchebycheffian B-splines*
16:15–17:00	Oleksandr Kozynenko: *Adaptive anisotropic approximation of multivariate functions by piecewise constants*
17:00–17:25	Ognyan Kounchev: *Non-parametric regression using smoothing polysplines for applications to Big Data*
Tuesday, February 20	
09:00–09:45	Tomas Sauer: *Hankel operators of finite rank*
09:45–10:10	Benedikt Diederichs: *Prony's problem and the restricted isometric property*
10:10–10:35	Wen-shin Lee: *Identification problems in multi-exponential analysis*
10:35–11:05	**Coffee break**
11:05–11:50	Michael Skrzipek: *Polynomials used in frequency analysis and their zeros*
15:00–15:30	**Coffee & cake**
15:30–16:15	Annie Cuyt: *From exponential analysis in signal processing to sparse interpolation, Padé approximation and tensor decomposition*
16:15–16:40	Brigitte Forster: *THE commutative diagram of signal processing in a Clifford setting*
Wednesday, February 21	
09:00–09:45	Bin Han: *Derivative-orthogonal multiwavelets and multiwavelets on the interval*
09:45–10:10	René Koch: *Analysis of shearlet coorbit spaces*
10:10–10:35	Daniela Schenone: *Edge detection methods based on RBF interpolation*
10:35–11:00	**Coffee break**
11:00–11:45	Johannes Wallner: *Material-minimizing forms and structures*
11:45–12:10	Armin Iske: *Kernel matrices with off-diagonal decay*
12:10–12:35	Matthias Beckmann: *Saturation rates for filtered back projection*
	Excursion
Thursday, February 22	
09:00–09:45	Jean-Louis Merrien: *Generalized Taylor operators and Hermite subdivision schemes*
09:45–10:10	Hartmut Prautzsch: *Mixed primal dual honeycomb schemes*
10:10–10:35	Svenja Hüning: *Polynomial reproduction of Hermite subdivision schemes*
10:35–11:00	**Coffee break**
11:00–11:25	Nira Dyn: *Smoothness of non-linear Lane-Reisenfeld algorithms refining numbers*
11:25–11:50	Sergio López-Ureña: *Self-adapting reproduction of trigonometric surfaces by non-linear subdivision*
15:00–15:30	**Coffee & cake**
15:30–16:15	Vladimir Protasov: *Regularity of multivariate refinable functions*
16:15–16:40	Ulrich Reif: *Moments of sets with refinable boundary*
16:40–17:05	Dörte Rüweler: *Filter banks for arbitrary dilations*
Friday, February 23	
09:00–09:30	*Packing and paying*
09:30–10:15	Ba-Duong Chu: *Maximum principle and elliptic PDEs*
10:15–10:40	Lars-Benjamin Maier: *Intrinsic elliptic PDE on submanifolds: approximate ambient solutions*
10:40–11:00	**Coffee break**
11:00–11:45	Florian Martin: *WEB-collocation for singular and time-dependent problems*
11:45–11:55	*Closing remarks*

Saturation rates for filtered back projection

Matthias Beckmann (University of Hamburg)

The method of filtered back projection (FBP) allows us to reconstruct bivariate functions from given Radon samples, where low-pass filters with finite bandwidth and compactly supported window functions are employed to make the reconstruction by FBP less sensitive to noise.

The aim of this talk is to analyse the inherent FBP reconstruction error which is incurred by the application of a low-pass filter. To this end, we present error estimates in Sobolev spaces of fractional order and provide quantitative criteria to a priori evaluate the performance of the utilized low-pass filter by means of its window function. The obtained error bounds depend on the bandwidth of the filter, on the flatness of its window function at the origin, on the smoothness of the target function, and on the order of the Sobolev norm. Further, we prove convergence for the approximate FBP reconstruction in the treated norms along with asymptotic convergence rates as the bandwidth goes to infinity, where we observe saturation at fractional order depending on smoothness properties of the window. The theoretical results are supported by numerical experiments.

This talk is based on joint work with Armin Iske.

`matthias.beckmann@uni-hamburg.de`

Maximum principle and elliptic PDEs

Ba-Duong Chu (Technical University Darmstadt)

The maximum principle is one of the most useful tools in the theory of partial differential equations. In particular, it can be used to obtain the error bounds between approximate solution and exact solution.
In this talk we present a combination of the so-called *hierarchical least-squares collocation* method and the maximum principle to provide pointwise lower and upper bounds for solutions of elliptic differential equations.

chu@mathematik.tu-darmstadt.de

From exponential analysis in signal processing to sparse interpolation, Padé approximation and tensor decomposition

Annie Cuyt (University of Antwerp), Wen-shin Lee*

What is the connection of tensor decomposition in multilinear algebra with exponential analysis from signal processing, sparse interpolation from computer algebra, Gaussian quadrature from numerical analysis, and Padé approximation theory? These seemingly unrelated and diverse topics are nevertheless deeply intertwined, as we explain here. However, several of these connections have remained unexplored.

We particularly focus on multi-exponential models

$$\Phi(x) = \sum_{j=1}^{n} \alpha_j \exp(\phi_j x), \qquad \alpha_j = \beta_j + \mathrm{i}\gamma_j, \qquad \phi_j = \psi_j + \mathrm{i}\omega_j,$$

representing signals which fall exponentially with time x.

A mathematical model is called n-sparse if it is a combination of only n generating elements. In sparse interpolation, the aim is to determine both the support of the sparse linear combination and the scalar coefficients in the representation, from a small or minimal amount of data samples. Sparse representations reduce the complexity in several ways: data collection, algorithmic complexity, model complexity.

In the past few years, insight gained from the computer algebra community combined with methods developed by the numerical analysis community, has lead to significant progress in several very practical and real-life signal processing applications. We make use of tools such as the singular value decomposition and various convergence results for Padé approximants to regularize this inverse problem.

annie.cuyt@uantwerpen.be

Prony's problem and the restricted isometric property

Benedikt Diederichs (University of Hamburg), Armin Iske*

In compressed sensing, one seeks to find the sparsest solution to an underdetermined linear system. In general, a sparse solution which is close to the measurements is not necessarily close to the ground truth. However, if the linear system satisfies a special property, the Restricted Isometric Property (RIP), this is indeed the case. This can be understood as a conditional well-posedness.

Prony's problem can also be seen as a sparse recovery problem, as one wishes to recover an exponential sum (which is sparse in the frequency domain). It is therefore interesting to see whether an analog of the RIP can be found, which guarantees that any two exponential sums fitting the model assumption and having close samples, always have similar frequencies. In this talk, we present such an analog of the RIP for Prony's problem.

benedikt.diederichs@uni-hamburg.de

Smoothness of non-linear Lane-Reisenfeld algorithms refining numbers

Nira Dyn (Tel-Aviv University), David Levin, Ron Goldman*

In this talk we discuss the smoothness of functions generated as limits of a Lane-Riesenfeld algorithm with the linear averages of numbers replaced by **different** non-linear averages of numbers. Sufficient conditions are given so that the smoothness equals that of the limit functios of the corresponding linear Lane-Riesenfeld algorithm. Our results extend those of Duchamp, Xie and Yu valid for uniform, non-linear Lane-Riesenfeld algorithms, and allow the scheme to be also non-uniform, but such that the non-linear averages change slowly with the location.

niradyn@post.tau.ac.il

THE commutative diagram of signal processing in a Clifford setting

Brigitte Forster (Universität Passau)

THE commutative diagram of signal processing consists of the Fourier Series, the Paley-Wiener theorem and the Sampling Theorem. In the L^2-setting, the theory is well known, for the classical case as well as for the case of non-harmonic Fourier series and entire functions of exponential type with various growth conditions. For $L^p(\mathbb{R})$-spaces, $p \neq 2$, the commutative diagram was established by Maergoiz in 2006. For entire functions with polygonal indicator diagram partial results are given by Levin/ Ljubarski in 1975 and Semmler/F. in 2015. Recently, the Clifford community developed Paley-Wiener theorems for multivariate functions on Clifford-Hilbert-spaces, see e.g. Kou/Tian 2002 and Franklin/Hogan/Larkin 2017. In the presentation, we show the developments of THE commutative diagram with respect of these aspects and give first results for the Clifford setting.

This is research in progress in collaboration with Jeff Hogan, University of Newcastle, Australia.

`post@brigitte-forster.de`

Derivative-orthogonal multiwavelets and multiwavelets on the interval

Bin Han (University of Alberta), M. Michelle*

Though wavelets for numerical computing have been extensively studied for many years, there are several key problems unresolved yet. Two of them are: (1) Riesz wavelets that are orthogonal with respect to derivatives at different scale levels for small condition numbers. (2) Wavelet bases on the interval $[0, 1]$ with simple structure to deal with different boundary conditions. In this talk we shall completely characterize derivative-orthogonal Riesz multiwavelets, present several examples using Hermite spline functions, and then apply them to the numerical solutions of differential equations. The development of derivative-orthogonal Riesz wavelets is closely linked to wavelets in general Sobolev spaces. As the second related part of this talk, we shall discuss how to construct wavelets and framelets on the interval $[0, 1]$ so that the wavelets can achieve different boundary conditions. This is a joint work with M. Michelle. The talk is based on the following work:

1. B. Han and M. Michelle, Derivative-orthogonal Riesz wavelets in Sobolev spaces with applications to differential equations, *Appl. Comput. Harmon. Anal.*, published online, (2017).

2. B. Han and M. Michelle, Construction of wavelets and framelets on a bounded interval, *Anal. Appl.*, in press, (2018)

3. B. Han, Framelets and Wavelets: Algorithms, Analysis, and Applications. *Applied and Numerical Harmonic Analysis*, Birkhäuser/Springer, Cham, (2017).

bhan@ualberta.ca

Polynomial reproduction of Hermite subdivision schemes

Svenja Hüning (TU Graz), Costanza Conti*

Subdivision algorithms produce limit curves by refining given sequences of input data. We say that a subdivision scheme has polynomial reproduction of degree d if for all polynomials of degree $\leqslant d$ it maps uniform samples of this polynomial to denser samples of the same polynomial. The case of point data is well-known: The mask of the scheme has to satisfy certain algebraic conditions in order to obtain polynomial reproduction.

In this presentation we deal with the Hermite case where data consists of values plus derivatives of a function. We give algebraic conditions on the mask of an Hermite subdivision scheme which characterise polynomial reproduction up to a certain degree.

huening@tugraz.at

Kernel matrices with off-diagonal decay

Armin Iske (University of Hamburg)

Kernel matrices are generated by radially symmetric positive definite kernel functions ϕ: For an ordered set $X = \{x_1, \dots, x_n\} \subset \mathbb{R}^d$ of pairwise distinct points the resulting kernel matrix is $A_{\phi,X} = \left(\phi(\left\|x_j - x_k\right\|_2)\right)_{1 \le j,k \le n} \in \mathbb{R}^{n \times n}$. For relevant kernels ϕ, e.g. for the Gaussians $\phi(r) = \exp(-r^2)$, a matrix entry $\phi(\left\|x - y\right\|_2)$ in $A_{\phi,X}$ is *small*, iff the Euclidean distance $\left\|x - y\right\|_2$ between $x, y \in X$ is *large*. In this talk, we show how to change the order of the points in X, by the application of a permutation σ on the indices of the points in X, such that for the resulting point set $\sigma(X) = \{x_{\sigma(1)}, \dots, x_{\sigma(n)}\}$ the kernel matrix $A_{\phi,\sigma(X)}$ has *off-diagonal decay*, i.e., such that matrix entries in $A_{\phi,\sigma(X)}$ close to the diagonal are large, whereas matrix entries far from the diagonal are small. To this end, we first discuss suitable measures to quantifying off-diagonal decay for kernel matrices, before we explain how to construct permutations σ, whose resulting kernel matrices $A_{\phi,\sigma(X)}$ have off-diagonal decay in situations of *small* dimensions d. Moreover, we show how to employ customized dimensionality reduction methods to obtain kernel matrices $A_{\phi,\sigma(X)}$ with off-diagonal decay also for *large* dimensions d. Supporting numerical examples are finally presented for illustration.

iske@math.uni-hamburg.de

Analysis of shearlet coorbit spaces

René Koch (RWTH Aachen University), Hartmut Führ*

Shearlet transforms have been introduced as class of directionally selective wavelet transforms. One way of describing the approximation-theoretic properties of such generalized wavelet systems relies on *coorbit spaces*, i.e., spaces defined in terms of sparsity properties with respect to the system. In higher dimensions, there are several distinct possibilities for the definition of shearlet systems, and their approximation-theoretic properties are currently not well-understood.

In this talk we investigate shearlet systems in higher dimensions derived from two particular classes of shearlet groups, the standard shearlet group and the toeplitz shearlet group. We want to show that different groups have different approximation theories. The analysis of the associated spaces relies on an alternative description via *decomposition spaces*. For a shearlet group $H \subset \mathrm{GL}(d, \mathbb{R})$, this identification is based on a covering of the dual orbit $H^{-t}\xi =: \mathcal{O} \subset \mathbb{R}^d$ ($\xi \in \mathbb{R}^d$ suitable) induced by the shearlet group. The geometry of the sets in this covering is the determining factor for the associated decomposition space. We will see that $\mathcal{O}$ can be equipped with a metric structure that encodes essential properties of this covering. The orbit map $p_\xi : H \to \mathcal{O}, h \mapsto h^{-t}\xi$ then allows to compare the geometric properties of coverings induced by different groups without the need to explicitly compute the respective coverings. Finally, we apply a rigidity theorem which states that *geometrically incompatible* coverings lead to different decomposition spaces in all cases.

rene.koch@mathA.rwth-aachen.de

Non-parametric regression using smoothing polysplines for applications to Big Data

Ognyan Kounchev (Bulgarian Academy of Sciences)

The smoothing splines and their generalizations in one dimension have been extensively studied since about 1970 by G. Wahba, B. Silverman, and many other authors. Fast algorithms have been developed for them, in particular the one by Reinsch.

For approximation of multivariate data one has considered different alternatives, as Thin Plate Splines, Kriging, etc. The development of fast algorithms for these models becomes very urgent due to the necessity to process large amounts of data. In the present research we provide a study of fast algorithms for models based on smoothing polysplines, the last being introduced in the monograph "Multivariate Polsyplines. Applications to Numerical and Wavelet Analysis", Acad. Press/Elsevier, 2001.

The present research has been carried out jointly with Ts. Tsachev and H. Render. It has been supported by NSF of Bulgaria, grant DH-02/13, and by the Alexander von Humboldt Foundation, Bonn.

kounchev@gmx.de

Adaptive anisotropic approximation of multivariate functions by piecewise constants

Oleksandr Kozynenko (Oles Honchar Dnipro National University), O. Davydov, D. Skorokhodov*

Let $\Omega \subset \mathbb{R}^d$, $d \geqslant 2$, be a bounded domain. A finite collection Δ of subdomains $\omega \subset \Omega$ is called a *partition* of Ω provided that $\omega \cap \omega' = \varnothing$, for any $\omega, \omega' \in \Delta$, $\omega \neq \omega'$, and $\sum\limits_{\omega \in \Delta} |\omega| = |\Omega|$, where $|\cdot|$ is the Lebesgue measure. We call a partition Δ *convex* if every cell $\omega \in \Delta$ is convex, and for $N \in \mathbb{N}$, denote by $\mathfrak{D}_N$ the set of all convex partitions of Ω comprising N cells.

For $1 \leqslant q \leqslant \infty$ and $k \in \mathbb{N}$, by $W_q^k(\Omega)$ we denote the standard Sobolev space of functions $f : \Omega \to \mathbb{R}$.

For a partition Δ of Ω, we denote by $\mathcal{S}(\Delta)$ the space of functions $s : \Omega \to \mathbb{R}$ constant on every $\omega \in \Delta$. For $1 \leqslant p \leqslant \infty$, we define the error of the best L_p-approximation of a function $f : \Omega \to \mathbb{R}$ by picewise constant functions on N cells: $E_N(f)_p := \inf\limits_{\Delta \in \mathfrak{D}_N} \inf\limits_{s \in S(\Delta)} \left\| f - s \right\|_{L_p(\Omega)}$.

It was established in [1] that for $f \in W_q^1(\Omega)$, the quantity $E_N(f)_p$ behaves as $O\left(N^{-1/d}\right)$ as $N \to \infty$ provided that $\frac{1}{d} + \frac{1}{p} - \frac{1}{q} > 0$. O. Davydov in [2] constructed an approximation method with anisotropic partitions allowing to improve the estimate of the order of $E_N(f)_p$ to $O\left(N^{-2/(d+1)}\right)$ as $N \to \infty$ for functions $f \in W_p^2(\Omega)$. He also indicated that this $\frac{2}{d+1}$ is the saturation order of piecewise constant approximation. In the current work we were able to estimate the order of $E_N(f)_p$ for functions $f \in W_q^2(\Omega)$ for a wide range of parameters p and q, and show that such approximation order can be achieved by a sort of greedy algorithms.

Theorem 1. *Let $\Omega \subset \mathbb{R}^d$ be a bounded domain, $1 \leqslant p \leqslant \infty$ and $1 \leq q \leq \infty$ be such that $\frac{2}{d+1} + \frac{1}{p} - \frac{1}{q} > 0$, and let $f \in W_q^2(\Omega)$. Then*

$$E_N(f)_p \leq C(d,p,q) N^{-\frac{2}{d+1}} \left(|f|^q_{W_q^1(\Omega)} + |f|^q_{W_q^2(\Omega)} \right)^{\frac{1}{q}},$$

with the constant $C(d,p,q)$ independent on f.

1. Birman M. S.,Solomyak M. Z. Piecewise polynomial approximation of functions of the classes W_p^α. Mat. Sb., 1967, 73 (115), no. 3, 331-355 (in Russian). English translation in Math. USSR-Sb., 1967, 2, no. 3, 295-317.
2. Davydov O. Approximation by piecewise constants on convex partitions. J. Approx. Theory, 164 (2012), 346-352.

kozinenkoalex@gmail.com

Identification problems in multi-exponential analysis

Wen-shin Lee (University of Antwerp), Annie Cuyt*

We consider the interpolation of a d-variate exponential sum

$$f(x_1, \dots, x_d) = \sum_{j=1}^{n} \alpha_j \exp(\phi_{j,1} x_1 + \cdots + \phi_{j,d} x_d).$$

In the univariate case, where $d = 1$, there is an entire branch of algorithms, which can be traced back to Prony's method dated in the 18th century and devoted to the recovery of the $2n$ unknowns, $\alpha_1, \dots, \alpha_n, \phi_1, \dots, \phi_n$ in

$$f(x) = \sum_{j=1}^{n} \alpha_j \exp(\phi_j x).$$

In the multivariate case, where $d > 1$, it remains an active research topic to identify and separate distinct multivariate parameters from results computed by a Prony-like method from samples along projections.

On top of the above, if the $\phi_{j,k}$ are allowed to be complex, the evaluations of the imaginary parts of distinct $\phi_{j,k}$ can also collide. This aliasing phenomenon can occur in either the univariate or the multivariate case.

Our method interpolates $f(x_1, \dots, x_d)$ from $(d+1) \cdot n$ evaluations. Since the total number of parameters α_j and $\phi_{j,k}$ is exactly $(d+1) \cdot n$, we interpolate $f(x_1, \dots, x_d)$ from the minimum possible number of evaluations. The method can also be used to recover the correct frequencies from aliased results. Essentially, we offer a scheme that can be embedded in any Prony-like algorithm, such as the least squares Prony version, ESPRIT, the matrix pencil approach, etc., thus can be viewed as a new tool offering additional possibilities in exponential analysis.

`wen-shin.lee@uantwerpen.be`

Self-adapting reproduction of trigonometric surfaces by non-linear subdivision

Sergio López-Ureña (Universitat de València.), Costanza Conti, Lucia Romani*

This talk deals with the capability of subdivision schemes to automatically reproduce trigonometric functions. In [1, 2, 5] algebraic conditions for the masks of level-dependent linear subdivision schemes to reproduce exponential polynomials were given. Since then, families of linear non-stationary subdivision schemes reproducing exponential polynomials were defined and analysed.

In this work we construct a non-linear subdivision scheme capable of reproducing bivariate functions that are defined on the three-directional grid of the plane and belong to the 5 dimensional space

$$V_\gamma = \text{span}\left\{1, \exp(\pm\gamma_1 z_1 \pm \gamma_2 z_2)\right\}, \qquad \gamma_1, \gamma_2 \in \mathbb{R} \cup \imath(-\pi, \pi).$$

The key property of this non-linear scheme is that, differently from the already existing linear schemes, it does not need a pre-knowledge of the exponential polynomials to be reproduced (i.e., of the γ_1, γ_2 values).

This result is obtained by the help of certain *annihilation operators* for trigonometric functions, which can be seen as the bivariate extension of the *orthogonal rules* used in [4]. Following the ideas of a previous univariate work [3], we here use annihilation operators to define the new bivariate non-linear subdivision scheme from a linear non-stationary one. The resulting subdivision scheme is shown to be convergent, C^1-regular and capable of reproducing piecewisely defined trigonometric functions as well, since pre-knowledge of γ_1, γ_2 is not needed.

Keywords: Subdivision scheme, non-linearity, exponential polynomial reproduction, 3-directional grid, proximity

Acknowledgments. Supported by the research project MTM2014-54388 (Ministry of Economy and Competitivity, MINECO, Spain) and the FPU14/02216 grant (Ministry of Education and Culture and Sports, MECD, Spain).

References

[1] Maria Charina, Costanza Conti, and Lucia Romani. Reproduction of exponential polynomials by multivariate non-stationary subdivision schemes with a general dilation matrix. *Numerische Mathematik*, 127(2):223–254, 2014.

[2] Costanza Conti and Lucia Romani. Algebraic conditions on non-stationary subdivision symbols for exponential polynomial reproduction. *Journal of Computational and Applied Mathematics*, 236(4):543–556, 2011.

[3] Rosa Donat and López-Ureña Sergio. A non-linear stationary subdivision scheme that reproduces trigonometric functions. *In preparation*.

[4] Nira Dyn, David Levin, and Ariel Luzzatto. Exponentials reproducing subdivision schemes. *Foundations of Computational Mathematics*, 3(2):187–206, 2003.

[5] Paola Novara, Lucia Romani, and Jungho Yoon. Improving smoothness and accuracy of modified butterfly subdivision scheme. *Applied Mathematics and Computation*, 272:64–79, 2016.

sergio.lopez-urena@uv.es

Tchebycheffian B-splines

Tom Lyche (University of Oslo)

We give a short survey of Tchebycheffian Splines and Tchebycheffian B-splines. These piecewise functions are natural generalizations of polynomial splines and polynomial B-splines. Each segment of the spline belongs to a piecewise Extended Complete Tchebycheff (ECT)-space. We discuss various ways to define them. And chose a recursive definition, allowing different spaces on the segments. Several examples will be given to illustrate the concepts.

tom@math.uio.no

Intrinsic elliptic PDE on submanifolds: approximate ambient solutions

Lars-Benjamin Maier (Techn. Univ. Darmstadt), Ulrich Reif*

We present a novel approach to the solution of intrinsic elliptic partial differential equations on embedded submanifolds, based on functional minimization in the ambient space. The method appeared as an extension to the authors previous results on energy minimization under point constraints in the context of sparse data extrapolation. It is capable of providing optimal convergence rates with respect to certain energy norms, also confirmed by numerical experiments.

maier@mathematik.tu-darmstadt.de

WEB-collocation for singular and time-dependent problems

Florian Martin (University of Stuttgart)

The collocation method with weighted extended B-splines (WEB-splines) represents a recently published approach for the spline approximation of the solution of stationary partial differential equations. In contrast to standard finite element methods, WEB-collocation requires no mesh generation and numerical integration, which leads to considerably faster computation times and an easier implementation.
In this talk, the basics of WEB-collocation for general boundary value problems with mixed boundary conditions are described and the advantages over Ritz-Galerkin methods are illustrated for Poisson's equation as typical model problem. On this basis, current research results from the application to singular and time-dependent problems are presented. The utilization of uniform spline spaces permits a straightforward generalization of the basic concept to hierarchical bases and the development of intuitive refinement strategies. The benefits of these adaptive WEB-collocation algorithms are shown in case of the model problem with a singular solution. Furthermore, considering the problem of simulating a tsunami, the combination of the WEB-colloction concept and a time-step iteration is presented to demonstrate a novel approximation scheme for time-dependent equations.

florian.martin@mathematik.uni-stuttgart.de

Generalized Taylor operators and Hermite subdivision schemes

Jean-Louis Merrien (INSA Rennes), Tomas Sauer*

For vector subdivision schemes (VSS) it is not so straightforward to prove more than the Hölder regularity of the limit function. On the other hand, Hermite subdivision schemes (HSS) produce function vectors that consist of derivatives of a certain function, so that the notion of convergence automatically includes regularity of the limit.

Previously, we had established an equivalence between a *spectral condition* or *sum rules* and operator factorizations that transform a HSS into a VSS for which analysis tools are available. It was even conjectured that the spectral condition might be necessary for convergence.

We define general spectral conditions that generalize the classical one. These spectral conditions are more or less equivalent to the existence of a factorization with respect to respective generalized Taylor operators.

Conversely, we construct HSS of arbitrary regularity with guaranteed convergence and, in particular, give examples of convergent schemes that do not satisfy the spectral condition.

`jmerrien@insa-rennes.fr`

Mixed primal dual honeycomb schemes

Hartmut Prautzsch (Karlsruhe Institute of Technology), Huining Meng*

Local corner cutting is well understood for curves, i. e., we know which corner cutting schemes generate C^1 curves. However, a similar result is not known for surfaces generated by local corner cutting. Moreover, we know of no corner cutting scheme for polyhedra except for one implicit corner cutting scheme: The honeycomb scheme by Dyn, Levin and Liu is an interpolatory convexity preserving refinement scheme for convex polyhedra generating C^1 surfaces in the limit. Thus there are tangent planes for all vertices and these define local cuts of the tangent polyhedron.

In general, the "honeycomb surfaces" have planar or just line segments. In this talk, we show how one can modify the honeycomb scheme to get rid of the line segments: (1) We generalize the honeycomb scheme and (2) dualize it. Thus we obtain an explicit corner cutting scheme generating limiting surfaces without line segments but which are not smooth in general. Therefore (3) we propose to apply both schemes alternately. As we show, such mixed primal dual honeycomb schemes or MPD-schemes generate smooth limiting surfaces without line segments.

prautzsch@kit.edu

Regularity of multivariate refinable functions

Vladimir Protasov (University of L'Aquila; Moscow State University), Maria Charina*

Formulas for Holder and Sobolev exponents of univariate compactly supported refinabe functions are well-known. Any generalization of those formulas to multivariate case offers a surprising resistance. The only case when such a generalization is known in the literature is when the dilation matrix is isotropic, i.e., is similar to an orthogonal matrix. The same can be said about the higher order regularity analysis. We present closed formulas for Holder, L_p, and Sobolev regularity of multivariate refinable functions in case of arbitrary dilation matrix. Some applications are discussed. In particular, we derive a formula for the rate of convergence of a subdivision scheme with an arbitrary dilation matrix.

v-protassov@yandex.ru

Moments of sets with refinable boundary

Ulrich Reif (TU Darmstadt), Jan Hakenberg*

We present a method for determining moments (volume, center of mass, inertia tensor, etc.) for objects which are bounded by refinable functions, such as 2d sets bounded by subdivision curves and 3d sets bounded by subdivision surfaces. The approach is based on the solution of an eigenequation resulting from the interrelation between a certain multilinear form and its refined counterparts.

`reif@mathematik.tu-darmstadt.de`

Filter banks for arbitrary dilations

Dörte Rüweler (University of Passau), Tomas Sauer*

To extract features from an image, in particular directed edges, one can use multiple filter banks with *anisotropic* dilation matrices. For that purpose, we consider theory and implementation of filter banks for arbitrary dilation matrices. The downsampling operation can be efficiently implemented based on the Smith factorization of the dilation matrix. After pointing out the difference between arbitrary factorizations and the normal form, we show some examples of how this affects the implementation of the downsampling operation.

doerte.rueweler@uni-passau.de

Hankel operators of finite rank

Tomas Sauer (University of Passau)

We consider generalized multivariate Hankel operators of the form

$$Hx = \sum_{\alpha \in \mathbb{Z}^s} h(\cdot + \alpha)\, x(\alpha)$$

which are of finite rank. It turns out that they are closely related to finite dimensional shift invariant spaces and, of course, to Prony's problem. The main connection is a description of filters via zeros of the symbol and their multiplicity.

Tomas.Sauer@uni-passau.de

Edge detection methods based on RBF interpolation

Daniela Schenone (Università degli Studi Milano-Bicocca), Lucia Romani, Milvia Rossini*

Edge detection is a widely used tool in signal/image processing with the aim of identifying abrupt changes or discontinuities in a signal/digital image. In [2] and [3] an iterative adaptive method based on RBF interpolation with suitable scale parameters has been proposed for the detection of jump discontinuities in 1D and 2D problems. Generalizing the 1D method using variably scaled kernels [1] we obtain a new iterative method that compares favorably with the existing one since performs a smaller number of iterations and detects jumps more accurately.
We also present a new non-iterative technique based on RBF interpolation, that detects jumps/edges by identifying the local maxima of the absolute values of the expansion coefficients. To illustrate the effectiveness and efficiency of this last method, numerical examples in 1D and 2D are provided.

References

[1] M. Bozzini, L. Lenarduzzi, M. Rossini, R. Schaback, Interpolation with variably scaled kernels, IMA J. Numer. Anal. 35(1) (2015) 199–219.

[2] J.-H. Jung, V. R. Durante, An iterative adaptive multiquadric radial basis function method for the detection of local jump discontinuities, Appl. Numer. Math., 59 (2009) 1449–1466.

[3] J.-H Jung, S. Gottlieb, S. O. Kim, Iterative adaptive RBF methods for detection of edges in two-dimensional functions, Appl. Numer. Math., 61 (2011) 77–91.

daniela.schenone@unimib.it

Polynomials used in frequency analysis and their zeros

Michael Skrzipek (Fernuniversität in Hagen)

In the context of the reconstruction of a signal from given samples, the subproblem of determining the frequencies leads to the construction of a polynomial. From its zeros the desired frequencies can be obtained by simple transformations.

Two approaches are widely used. One calculates a (finite) sequence of Szegő polynomials $(s_\nu)_{\nu=0}^m$. The zeros of s_m are eigenvalues of a Hessenberg matrix, which is build up by the reflection coefficients of the Szegő polynomials.

Another classical approach goes back on 1795 and uses an idea of Baron de Prony to construct a polynomial (the 'Prony polynomial') in monomial representation. Its zeros can be calculated by standard numerical methods.

In our talk we show some connections between these approaches. Both methods depend very sensitive on sampling errors. But meanwhile, there exist some variants of these approaches which are less sensitive in their numerical behaviour and better suited for noised samples.

Characteristics of a signal reflect in properties of the corresponding Szegő- or Prony-like polynomial. Reasons for possible numerical difficulties can be detected, too. Although there exist many algorithms to calculate zeros of polynomials, we use the knowledge of such characteristics for the construction of particularly suitable algorithms to get the desired frequencies.

`michael.skrzipek@fernuni-hagen.de`

Material-minimizing forms and structures

Johannes Wallner (TU Graz), D. Pellis, M. Kilian, and H. Pottmann*

Three-dimensional structures in building construction and architecture are realized with conflicting goals in mind: engineering considerations and financial constraints easily are at odds with creative aims. It would therefore be very beneficial if optimization and side conditions involving statics and geometry could play a role already in early stages of design, and could be incorporated in design tools in an unobtrusive and interactive way. This paper, which is concerned with a prominent class of structures, is a substantial step towards this goal. We combine the classical work of Maxwell, Michell, and Airy with differential-geometric considerations and obtain a geometric understanding of "optimality" of surface-like lightweight structures. It turns out that total absolute curvature plays an important role. We enable the modeling of structures of minimal weight which in addition have properties relevant for building construction and design, like planar panels, dominance of axial forces over bending, and geometric alignment constraints.

`j.wallner@tugraz.at`

Speakers

List of participants

1. *Matthias Beckmann*, University of Hamburg, matthias.beckmann@uni-hamburg.de
2. *Maria Charina*, University of Vienna, maria.charina@univie.ac.at
3. *Costanza Conti*, Università di Firenze, costanza.conti@unifi.it
4. *Mariantonia Cotronei*, Università Mediterranea di Reggio Calabria, mariantonia.cotronei@unirc.it
5. *Ba-Duong Chu*, Technical University Darmstadt, chu@mathematik.tu-darmstadt.de
6. *Annie Cuyt*, University of Antwerp, annie.cuyt@uantwerpen.be
7. *Benedikt Diederichs*, University of Hamburg, benedikt.diederichs@uni-hamburg.de
8. *Nira Dyn*, Tel-Aviv University, niradyn@post.tau.ac.il
9. *Brigitte Forster*, Universität Passau, post@brigitte-forster.de
10. *Bin Han*, University of Alberta, bhan@ualberta.ca
11. *Svenja Hüning*, TU Graz, huening@tugraz.at
12. *Armin Iske*, University of Hamburg, iske@math.uni-hamburg.de
13. *René Koch*, RWTH Aachen University, rene.koch@mathA.rwth-aachen.de
14. *Ognyan Kounchev*, Bulgarian Academy of Sciences, kounchev@gmx.de
15. *Oleksandr Kozynenko*, Oles Honchar Dnipro National University, kozinenkoalex@gmail.com
16. *Wen-shin Lee*, University of Antwerp, wen-shin.lee@uantwerpen.be
17. *Sergio López-Ureña*, Universitat de València., sergio.lopez-urena@uv.es
18. *Tom Lyche*, University of Oslo, tom@math.uio.no
19. *Lars-Benjamin Maier*, Techn. Univ. Darmstadt, maier@mathematik.tu-darmstadt.de
20. *Florian Martin*, University of Stuttgart, florian.martin@mathematik.uni-stuttgart.de
21. *Jean-Louis Merrien*, INSA Rennes, jmerrien@insa-rennes.fr
22. *Hartmut Prautzsch*, Karlsruhe Institute of Technology, prautzsch@kit.edu
23. *Vladimir Protasov*, University of L'Aquila; Moscow State University, v-protassov@yandex.ru
24. *Ulrich Reif*, TU Darmstadt, reif@mathematik.tu-darmstadt.de
25. *Guenther Reitberger*, University of Passau, reitberg@forwiss.uni-passau.de
26. *Dörte Rüweler*, University of Passau, doerte.rueweler@uni-passau.de
27. *Tomas Sauer*, University of Passau, Tomas.Sauer@uni-passau.de
28. *Daniela Schenone*, Università degli Studi Milano-Bicocca, daniela.schenone@unimib.it
29. *Michael Skrzipek*, Fernuniversität in Hagen, michael.skrzipek@fernuni-hagen.de
30. *Michael Stock*, Universität Passau, FORWISS, stock@forwiss.uni-passau.de
31. *Alberto Viscardi*, Università degli studi di Milano-Bicocca, alberto.viscardi@unimib.it
32. *Johannes Wallner*, TU Graz, j.wallner@tugraz.at