2nd IM-Workshop on

Applied Approximation, Signals and Images

Bernried (Germany)
February 27-March 3, 2017

ABSTRACTS

Organizers:
Costanza Conti
Mariantonia Cotronei
Nira Dyn
Brigitte Forster
Tomas Sauer

Monday, February 27	
09:30–09:40	*Welcome & Opening*
09:40–10:30	Getting to know each other: Mathematical speed dating I
10:30–11:00	**Coffee break**
11:00–12:00	Getting to know each other: Mathematical speed dating II
15:00–15:30	**Coffee & cake**
15:30–16:15	De Marchi: *Lissajous sampling and adaptive spectral filtering for the reduction of the Gibbs phenomenon in Magnetic Particle Imaging*
16:15–17:00	Massopust: *Self-referentiality, fractels, and applications*
17:00–17:25	Schenone: *Automatic spline fitting of planar curvilinear profiles in digital images using the Hough transform*
Tuesday, February 28	
09:00–09:45	Führ: *The classification of anisotropic Besov spaces*
09:45–10:10	Koch: *Analysis of shearlet coorbit spaces in dimension three*
10:10–10:35	Gütschow: *Roulet transform (rotational anisotropic wavelet transformation)*
10:35–11:05	**Coffee break**
11:05–11:50	Elisha: *Wavelet decompositions of random forests*
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15:30–15:55	Pitolli: *A class of non-stationary biorthogonal wavelet filters*
15:55–16:20	Viscardi: *Semi-regular wavelet tight frames: a fail of the UEP*
16:20–16:45	Beckmann: *Error analysis for filtered back projection reconstructions fractional Sobolev spaces*
16:45–17:10	Boßmann: *Generalized orthogonal matching pursuit for multiple measurements*
17:10–17:35	Reif: *Automatic differentiation with Matlab*
Wednesday, March 1	
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09:45–10:10	Diederichs: *Improved estimates for condition numbers of RBF interpolation matrices*
10:10–10:35	Iske: *Scattered data approximation by weighted kernels*
10:35–11:00	**Coffee break**
11:00–11:45	Lorenz: *G^2-continuity and patches on surface transitions*
11:45–12:10	Sande: *Optimal spline spaces of higher degree for L^2 n-widths*
12:10–12:35	Kounchev: *A new cubature formula for functions with singularity in the disc, with error bound*
	Excursion
Thursday, March 2	
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09:45–10:10	Moosmüller: *A smoothing procedure for Hermite subdivision schemes*
10:10–10:35	Lipovetsky: *A weighted binary average of point-normal pairs in 2D and 3D with application to subdivision schemes*
10:35–11:00	**Coffee break**
11:00–11:25	Turati: *Multivariate pseudo-splines and multigrid*
11:25–11:50	Volontè: *Geometric conditions for curvature continuity of interpolatory planar subdivision curves*
15:00–15:30	**Coffee & cake**
15:30–15:55	Romani: *G^1-continuity of non-stationary subdivision schemes at the limit points of extraordinary vertices and faces*
15:55–16:20	Hüning and Wallner: *Contractivity and convergence of refinement schemes in Riemannian geometry*
16:20–16:45	López-Ureña: *New non-linear stationary subdivision scheme with trigonometric functions reproduction*
16:45–17:10	Mejstrik: *Joint spectral radius and multiple subdivision*
Friday, March 3	
09:15–10:00	Floater: *Polynomial interpolation on interlacing rectangular grids*
10:00–10:25	Cirillo: *Barycentric rational Hermite interpolation with no poles and high rates of approximation*
10:25–10:50	**Coffee break**
10:50–11:35	Sauer: *A tale of couples and other syzygies*
11:35–11:45	*Closing remarks*

Error analysis for filtered back projection reconstructions fractional Sobolev spaces

Matthias Beckmann, Armin Iske*

This talk concerns the approximation of bivariate functions by using the well-established filtered back projection (FBP) formula from computerized tomography, which allows us to reconstruct a bivariate function from given Radon data. The FBP formula, however, is numerically unstable. Therefore, suitable low-pass filters of finite bandwidth and with a compactly supported window function are employed to make the reconstruction by FBP less sensitive to noise.

The objective of this talk is to analyse the inherent FBP approximation error which is incurred by the application of the chosen low-pass filter. To this end, we present error estimates in Sobolev spaces of fractional order. The obtained error bounds depend on the bandwidth of the low-pass filter, on the flatness of the filter's window function at the origin, on the smoothness of the target function, and on the order of the considered Sobolev norm. Finally, we prove convergence for the approximate FBP reconstruction in the treated Sobolev norms along with asymptotic convergence rates, as the filter's bandwidth goes to infinity.

Generalized orthogonal matching pursuit for multiple measurements

Florian Boßmann

Orthogonal matching pursuit (OMP) is a greedy algorithm that seeks to find a sparse solution of the linear system

$$Ax = b,$$

i.e., a solution x containing only a small number of non-zero entries. These solutions are of special interest in applications where the measured data b is influenced by only a few (unknown) entries of the parameter vector x. However, in many applications not only one data vector b, but multiple measurements b_i, $i = 1, \dots, n$ are given. Now, the linear system

$$AX = B$$

has to be solved; here X and B are matrices. Still, we can assume that each column of X is sparse. Moreover, similar measurements will generate correlated vectors b_i, $i = 1, \dots, n$. Thus, we assume that the matrix X is structured. Generalized orthogonal matching pursuit for multiple measurements is an algorithm designed to capture the underlying structure. This way, the approximation quality of the solution can be enhanced and efficient post-processing of the data is permitted.

Barycentric rational Hermite interpolation with no poles and high rates of approximation

Emiliano Cirillo

In this talk we study an iterative approach to the Hermite interpolation problem, which first constructs an interpolant of the function values at $n+1$ nodes and then successively adds m correction terms to fit the data up to the m-th derivative. In the case of polynomial interpolation, this simply reproduces the classical Hermite interpolant, but the approach is general enough to be used in other settings. In particular, we focus on the family of barycentric rational Floater–Hormann interpolants, which are based on blending local polynomial interpolants of degree d with rational blending functions. For this family, the proposed method results in rational Hermite interpolants of degree $(m+1)(n+1)-1$, which converge at the rate of $O(h^{(m+1)(d+1)})$ as the mesh size h converges to zero.

Lissajous sampling and adaptive spectral filtering for the reduction of the Gibbs phenomenon in Magnetic Particle Imaging

Stefano De Marchi, Wolfgang Erb, Francesco Marchetti*

Polynomial interpolation and approximation methods on sampling points along Lissajous curves using Chebyshev series is an effective way for a fast image reconstruction in Magnetic Particle Imaging (MPI). Due to the nature of spectral methods, a Gibbs phenomenon occurs in the reconstructed image if the underlying function has discontinuities. A possible solution for this problem are spectral filtering methods acting on the coefficients of the approximating polynomial.

In this work, we introduce Lissajous sampling and classical filtering techniques in one and several dimensions. We then present an adaptive spectral filtering process for the reduction of the Gibbs phenomenon and for an improved approximation of the underlying function or image. In this adaptive filtering technique, the spectral filter depends on the distance of a spatial point to the nearest discontinuity. We show the effectiveness of this filtering approach in theory, in numerical simulations as well as in the application in Magnetic Particle Imaging.

Improved estimates for condition numbers of RBF interpolation matrices

Benedikt Diederichs

Interpolation by radial basis functions is a classic topic in multivariate approximation with many applications. In this problem, one encounters linear systems with the kernel matrices of the radial basis functions and it is of some interest to have precise estimates for their condition number. Therefore, estimates have been developed, to obtain bounds on the condition number in terms of the separation radius of the interpolation points. We present new estimates, which build upon extremal Fourier functions. We show that they are very close to optimal.

Wavelet decompositions of random forests

*Oren Elisha**, *Shai Dekel*

In this talk we introduce, in the setting of machine learning, a generalization of wavelet analysis which is a popular approach to low dimensional structured signal analysis. The wavelet decomposition of a Random Forest provides a sparse approximation of any regression or classification high dimensional function at various levels of detail, with a concrete ordering of the Random Forest nodes: from 'significant' elements to nodes capturing only 'insignificant' noise. Motivated by function space theory, we use the wavelet decomposition to compute numerically a 'weak-type' smoothness index that captures the complexity of the underlying function. As we show through extensive experimentation, this sparse representation facilitates a variety of applications such as improved regression for difficult datasets, a novel approach to feature importance, resilience to noisy or irrelevant features, compression of ensembles, etc. the talk is based on [1].

References

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Polynomial interpolation on interlacing rectangular grids

Michael S. Floater

In this talk we review some of the remarkable properties of Padua points and related point sets consisting of pairs of interlacing rectangular grids.

In particular, Padua points, defined in the domain $[-1, 1]^2$, are unisolvent for polynomial interpolation of full degree N. The Lebesgue constant grows with minimal order $O(\log^2(N))$ and the associated cubature rule has degree of precision $2N - 1$ with respect to the Chebyshev weighting. Similar properties have been established by Morrow and Patterson, Xu, and Erb et al. for similar pairs of interlacing rectangular grids, with respect to suitable spaces of polynomials that are no longer full, but in some cases close to full. In all these grids, the points have some kind of Chebyshev spacing in each coordinate direction.

We will then go on to focus purely on unisolvence and study the unisolvence of interlacing pairs of rectangular grids in which the spacing of the points in each coordinate direction is arbitrary. We will break this problem down using a combination of tensor-product interpolation, Newton interpolation, and a property of divided difference matrices.

The classification of anisotropic Besov spaces

Hartmut Führ

The homogeneous anisotropic Besov spaces $\dot{B}^{\alpha}_{p,q}(A)$ are defined in terms of wavelet coefficient decay over a wavelet system $(D_{A^j} T_k \psi)_{j \in \mathbb{Z}, k \in \mathbb{Z}^d} \subset \mathrm{L}^2(\mathbb{R}^d)$, with suitably chosen mother wavelet ψ. Here the dilation matrix A is chosen to be *expansive*, i.e., all eigenvalues of A have modulus > 1. For $A = 2 \cdot \mathbf{E}_d$, the Besov spaces are just the usual, isotropic ones. Prior to the work presented here, the dependence of the scale of spaces $\dot{B}^{p,q}_{\alpha}(A)$ on the choice of the dilation matrix was poorly understood.

In the talk, I completely characterize when two expansive matrices induce the same scale of anisotropic Besov spaces, and I answer the analogous question for the related case of inhomogeneous anisotropic Besov spaces. While these results are of independent interest, their proof can also be seen as a case study in the use of decomposition spaces for the investigation of function spaces, and the talk aims at explaining these aspects.

Very briefly, decomposition spaces are smoothness spaces whose norms quantify Fourier decay. The chief ingredient in the definition of a scale of decomposition spaces is an underlying covering of the frequency space. Recent results due to Voigtlaender allow to decide when two frequency coverings yield the same scale of decomposition spaces, by directly comparing the coverings. This approach can be brought to bear on the case of anisotropic Besov spaces, where it leads to a complete classification of expansive matrices. In a similar spirit, the related talk by René Koch uses the decomposition space approach to compare the approximation-theoretic properties of different shearlet constructions in dimension three.

Roulet transform

Rotational anisotropic wavelet transformation

Silja Gütschow

Most integral transforms, like the Wavelet- or Shearlet Transform, are induced by the quasi-regular representation of semidirect products. It is mostly known under which conditions these types of transforms detect wavefront sets of tempered distributions. Here this theory will be extended to a new transform, the *Roulet Transform*. The properties of this transform will be presented.

One possible application for the Roulet Transform is the detection of chatter marks in automotive fabrication.

Contractivity and convergence of refinement schemes in Riemannian geometry

Svenja Hüning, Johannes Wallner

We are interested in the convergence analysis of subdivision schemes in Riemannian geometry which are algorithms producing limit curves by refining discrete sets of points. So far, convergence results could be proven for certain classes of refinement rules and/or special kinds of Riemannian manifolds. Examples are interpolatory subdivision schemes and schemes with only positive mask coefficients. Another approach to obtain convergence statements is to show convergence only for 'dense enough' input data using so-called proximity conditions.

In this talk we extend a known convergence result (which applies to all input data) to refinement schemes whose mask contains negative coefficients. For that purpose we study the Riemannian center of mass on Cartan-Hadamard manifolds and prove a contractivity condition depending only on the mask coefficients of the subdivision scheme.

Scattered data approximation by weighted kernels

Armin Iske

Radial kernels are popular tools for multivariate scattered data approximation, where the utility of kernel-based reconstructions from generalized Hermite-Birkhoff data has been demonstrated in many applications. The approximation of images from scattered Radon data is only one relevant example. As we show, however, standard kernel-based reconstruction methods fail to work for this particular application. Therefore, we first explain limitations of radial kernels, before we propose *weighted* positive definite kernels, which are symmetric but not radially symmetric. We discuss the characterization and construction of weighted positive definite kernels in general, before we provide concrete examples. This leads us to a larger class of flexible kernel-based approximation schemes, which work for image reconstruction from scattered Radon data and other relevant applications.

Analysis of shearlet coorbit spaces in dimension three

*Hartmut Führ, René Koch**

Shearlet groups have received much attention lately since their associated shearlet transform is superior to the usual wavelet approach in the representation and encoding of anisotropic features of multidimensional data. These groups can be used to define classes of certain smoothness spaces, called shearlet coorbit spaces. Coorbit spaces can be understood as spaces of signals which can be well approximated by the shearlet system, and the smoothness of their elements corresponds to the decay of an associated transform. We will consider shearlet groups in dimension three with the following structure

$$\left\{ \pm \begin{pmatrix} a & ab & ac \\ 0 & a^{\lambda_1} & 0 \\ 0 & 0 & a^{\lambda_2} \end{pmatrix} : \begin{matrix} a > 0 \\ b, c \in \\ \mathbb{R} \end{matrix} \right\} \quad \text{and} \quad \left\{ \pm \begin{pmatrix} a & ab & ac \\ 0 & a^{1-\delta} & a^{1-\delta} b \\ 0 & 0 & a^{1-2\delta} \end{pmatrix} : \begin{matrix} a > 0 \\ b, c \in \\ \mathbb{R} \end{matrix} \right\}$$

for $\lambda_1, \lambda_2, \delta \in \mathbb{R}$ and examine their associated coorbit spaces.

The study and comparison of shearlet coorbit spaces associated to different shearlet groups relies on an alternative description of these spaces as decomposition spaces. This approach allows us to identify which properties of the groups are decisive for the structure of the related coorbit spaces and is an example for the application of a decomposition space viewpoint to investigate function spaces.

This talk serves as a case study in the systematic application of decomposition spaces for the investigation of smoothness spaces. We will see that shearlet coorbit spaces with respect to different shearlet groups are in almost all cases isomorphic to different decomposition spaces.

A new cubature formula for functions with singularity in the disc, with error bound

Ognyan Kounchev

We construct a new cubature formula for the disc which is applicable to functions having singularity in the disc. The crux of the formula is that we are able to write an error bound for a wide class of functions with singularities, and this error bound is completely constructive. In a certain sense, our formula and the error bound are a generalization of the one-dimensional quadrature formula of Gauss-Jacobi where the singularities are lying on the boundary of the interval. The main results have been announced in https://arxiv.org/abs/1509.00283 and https://arxiv.org/abs/1509.00060 .

Attractors of sequences of function systems and the relation to non-stationary subdivision

David Levin

Iterated Function Systems (IFSs) have been at the heart of fractal geometry almost from its origin, and several generalizations for the notion of IFS have been suggested. Subdivision schemes are widely used in computer graphics and attempts have been made to link limits generated by subdivision schemes to fractals generated by IFSs. With an eye towards establishing connection between non-stationary subdivision schemes and fractals, this talk introduces the notion of "trajectories of maps defined by function systems" which may be considered as a new generalization of the traditional IFS. The significance and the convergence properties of 'forward' and 'backward' trajectories is presented. Unlike the ordinary fractals which are self-similar at different scales, the attractors of these trajectories may have different structures at different scales.

This is a joint work with Nira Dyn, Tel Aviv University, and Puthan Veedu Viswanathan, IIT Delhi.

A weighted binary average of point-normal pairs in 2D and 3D with application to subdivision schemes

Evgeny Lipovetsky

Subdivision is a well-known and established method for generating smooth curves/surfaces from discrete data by repeated refinements. The typical input for such a process is a mesh of vertices. In this talk we propose to refine 2D or 3D data consisting of vertices of a polygon and a normal at each vertex. We first present our core refinement procedure - the 2D circle average, which is a non-linear weighted average of two points and their corresponding normals. We modify linear subdivision schemes refining points, to schemes refining point-normal pairs, by replacing the weighted binary arithmetic means in a linear subdivision scheme, by circle averages with the same weights. We investigate the so modified Lane-Riesenfeld algorithm and the 4-point scheme. We show C1 smoothness of the limit curves in 2D for each of these modified schemes.

Next, a generalization of the circle average to 3D is presented and some experimental results are demonstrated in 3D.

This is work is part of my research towards Ph.D under the supervision of Nira Dyn.

New non-linear stationary subdivision scheme with trigonometric functions reproduction

*Rosa Donat, Sergio López-Ureña**

Reproduction of trigonometric functions is an interesting property for some applications of subdivision schemes, like in CAGD. The reproduction of exponential polynomials, which is a more general space, is a depth studied property [1,3], and we can achieve it using linear non-stationary subdivision schemes.

There is a unique four points linear and non-stationary subdivision scheme reproducing

$$\mathcal{F}(\gamma) := \text{span}\{1, \exp(i\gamma t), \exp(-i\gamma t)\} = \text{span}\{1, \cos(\gamma t), \sin(\gamma t)\}, \qquad i = \sqrt{-1},$$

for each γ value. However, we need to know γ to define such subdivision scheme. In this talk we show how to define a four-points non-linear and stationary subdivision scheme reproducing $\mathcal{F}(\gamma)$ which does not depend of γ. Consequently, this non-linear subdivision scheme reproduces $\mathcal{F}(\gamma)$ for several γ values.

Moreover, we can proof that it is convergent, monotonicity preserving and, in monotone zones, it has forth order of approximation *after-one-step*. Using the *proximity* theory [2,4], we can also proof its stability, smoothness and the approximation order.

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G^2-continuity and patches on surface transitions

Florian Lorenz

The talk will address the C^n/G^n-continuity of surface transitions in CAD-systems. By means of least squares it is possible to develop stable algorithms to evaluate the continuity quantitatively even at almost singular points. Moreover, an algorithm will be presented to join two surface patches with G^2-continuity by using appropriate shape-parameters and minimizing an energy functional of the form

$$\int\limits_{[F_u]}\int\limits_{[F_v]} F_{uu}(u,v)_2^2 + 2F_{uv}(u,v)_2^2 + F_{vv}(u,v)_2^2 dv du.$$

Scattered data approximation on submanifolds: (combined) ambient approaches

Lars-Benjamin Maier

The talk will cover novel approaches to certain problems of approximation theory on submanifolds both with and without boundary. An emphasis will be set on certain aspects of scattered data approximation in various forms and on functional minimization techniques in ambient and intrinsic settings.
We will present convergence behaviour and practical results for sparse, dense, locally sparse and clustered scattered data approximation problems, including techniques for filling holes in data and approaches for local refinement. We will especially provide enhanced results on optimal convergence of the Ambient B-Spline-Method (ABM), thereby verifying an open conjecture on that question.

Self-referentiality, fractels, and applications

Peter Massopust

We introduce the concept of self-referentiality and fractels for functions and discuss their analytic and algebraic properties. We also consider the representation of polynomials and analytic functions using fractels, and the consequences of these representations in numerical analysis.

Joint spectral radius and multiple subdivision

Thomas Mejstrik

Multivariate multiple subdivision schemes were introduced and analysed in [3] with applications discussed in [2]. Multiple Subdivision is a generalization of non-stationary subdivision with finitely many masks of bounded support and varying dilations. In this talk, we unify the convergence analysis from [3] with the convergence analysis from [1]. In particular we show the JSR-approach is applicable in the concept of multiple subdivision and that the characterization of convergence from Sauer are useful for the characterization of convergence in the non-stationary setting.

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A Smoothing Procedure for Hermite Subdivision Schemes

Caroline Moosmüller

In this talk we study the regularity of curves generated by Hermite subdivision schemes. In particular, we are interested in increasing this regularity. In scalar subdivision, it is well known that a scheme which produces C^ℓ limit curves can be transformed to a new scheme producing $C^{\ell+1}$ limit curves by multiplying the scheme's symbol with the *smoothing factor* $\frac{z+1}{2}$. We present a similar smoothing procedure for Hermite subdivision schemes, approaching this problem algebraically by manipulating the symbol of a given scheme. The algorithm presented in this talk allows to construct Hermite subdivision schemes of arbitrarily high regularity from Hermite schemes whose Taylor scheme is at least C^0.

This talk is based on a joint work with Nira Dyn.

A class of non-stationary biorthogonal wavelet filters

*Vittoria Bruni, Mariantonia Cotronei, Francesca Pitolli**

We analyze the properties of a family of non-stationary biorthogonal wavelet systems and investigate their use in signal and image processing methods. Particular attention is devoted to their ability in compacting image information (image sparsification). In fact, the good trade-off between number of vanishing moments and support length of the involved filters emphasizes the persistency property of the wavelet coefficients along scales allowing a better detection of singularity points in piecewise regular signals. The performance of such non-stationary filters in classical image processing problems, e.g. image compression or edge localization, is illustrated.

Automatic differentiation with Matlab

Ulrich Reif

Automatic differentiation is a technique to compute derivatives of functions by means of operator overloading. Unlike finite differences, it is exact up to round-off errors, and unlike symbolic differentiation, is is able to differentiate the outcome of algorithms, almost independent of their complexity. We present an implementation of automatic differentiation under Matlab and discuss some of its features and limitations.

G^1-continuity of non-stationary subdivision schemes at the limit points of extraordinary vertices and faces

Lucia Romani

In this talk we generalize the theoretical results published in [1].
Specifically, we provide a general criterion to establish convergence and G^1-continuity of a bivariate non-stationary subdivision scheme when the initial polygonal mesh contains extraordinary vertices and/or extraordinary faces.
This work is in collaboration with Costanza Conti (University of Florence), Marco Donatelli and Paola Novara (University of Insubria, Como).

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Optimal spline spaces of higher degree for L^2 n-widths

*Michael S. Floater, Espen Sande**

Building on previous work by Melkman and Micchelli, we will discuss how one can derive optimal subspaces for Kolmogorov n-widths in the L^2 norm with respect to sets of functions defined by kernels. This enables us to prove the existence of optimal spline subspaces of arbitrarily high degree for certain classes of functions in Sobolev spaces of importance in finite element methods. We construct these spline spaces explicitly in special cases.

References

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A tale of couples and other syzygies

Tomas Sauer

In contrast to the univariate case, finding point configurations in $\mathbb{R}^s$ that allow for unique interpolation by polynomials of total degree at most n is becoming nontrivial for $s \geq 2$, being not a matter of counting any more, but a geometric issue, cf. [5]. Early on people developed methods to construct such "correct" sets, the two most well–known one based on intersections of lines are due to Radon–Berzolari [1] as well as the *geometric characterization* GC_n of degree n due to Chung and Yao [2]. In 1982 [4] Gasca and Maeztu conjectured that all bivariate point sets satisfying the GC_n condition are in fact the outcome of the Radon–Berzolari construction. So far, this conjecture has been proved by mainly combinatoric arguments up to degree $n = 5$, [6]. Recently, Hal Schenck [3] pointed out an approach based on substantial algebraic geometry, in particular the Hilbert–Burch theorem, that relates the Gasca–Maeztu conjecture to properties of a certain matrix of syzygies.

This talk, based on joint work with Jesus Carnicer (Zaragoza) surveys the concepts and gives an elementary approach to the syzygy matrix and its meaning for the existence of so-called *maximal lines*.

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Automatic spline fitting of planar curvilinear profiles in digital images using the Hough transform

Daniela Schenone

The Hough transform is a well-established technique used in image analysis and digital image processing to recognize shapes in images with noisy backgrounds. One of the drawbacks of this technique is the need to identify a potentially approximating family of curves before the recognition algorithm can start.

The goal of this talk is thus to develop an innovative procedure for the automated recognition of both closed and open curvilinear profiles in 2D digital images, without knowing neither a family of predefined curves nor a predefined look-up table of a prototypal shape. Our method provides a G^1 continuous spline curve -eventually containing C^0 junctions where cusps occur- which approximates the sought profile. Moreover, as in the case of the standard Hough transform, the developed method retains robustness with respect to background noise.

This work is in collaboration with Costanza Conti (University of Florence) and Lucia Romani (University of Milano-Bicocca, Italy).

Multivariate pseudo-splines and multigrid

Valentina Turati

In this talk we introduce a new family of multivariate pseudo-splines with anisotropic dilation. We analyze their generation/reproduction properties and convergence. We explain the connection between anisotropic subdivision and multigrid and apply our results for numerical solution of anisotropic Laplacian.

This work is in collaboration with Maria Charina (University of Vienna), Marco Donatelli (University of Insubria - Como) and Lucia Romani (University of Milano-Bicocca).

Semi-regular wavelet tight frames: a fail of the UEP

Alberto Viscardi

Ron and Shen's *unitary extension principle* (UEP [4] [3]) is a well known tool for the construction of wavelet tight frames in the shift-invariant setting and is based on factorization of trigonometric polynomials. In [1] and [2], Chui, He and Stöckler developed a generalization of UEP for the non-shift-invariant setting that recasts the problem into a symmetric factorization of $S - PSP^T$, where P is a refinement matrix and S is an approximation of the inverse Gramian of the family of P-refinable functions. For the B-spline case $S - PSP^T$ is positive semi-definite and always leads to a wavelet tight frame with one vanishing moment. In this talk I will focus on semi-regular subdivisions defined on the knot partition $-h_1\mathbb{N} \cup \{0\} \cup h_2\mathbb{N}$ with $h_1, h_2 \geq 0$. If we consider the first order approximation of the inverse Gramian we get $S = I$, which corresponds to the standard UEP. However, there exist subdivision schemes for which $I - PP^T$ is indefinite and, thus, the UEP construction fails.

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Geometric conditions for curvature continuity of interpolatory planar subdivision curves

*Kai Hormann, Elena Volontè**

Subdivision schemes are an iterative process for generating curves, that are smooth up to pixel accuracy. The most common subdivision schemes used in computer graphics are linear schemes where at each iteration the new points are a linear combination of the points from the previous step. Linear schemes are well investigated because linearity allows to study easily the convergence and smoothness of the limit curve. Differently, in this presentation we concentrate on non-linear schemes, where new points are computed via geometric construction. Such schemes generate limit curves that take into account the geometry of the starting control polygon. To study convergence and smoothness in this case we substitute analytic continuity C^n with geometric continuity G^n, which means we consider tangents instead of first derivatives and curvature instead of second derivatives. In this talk we present sufficient conditions, which guarantee that a generic interpolatory non-linear subdivision scheme converges and gives G^2-continuous limit curves.

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