

IM-Workshop on
Signals, Images, and Approximation

Bernried (Germany)
February 29 – March 4, 2016

ABSTRACTS

Organizers:

Costanza Conti
Mariantonia Cotronei
Nira Dyn
Brigitte Forster
Tomas Sauer

Monday, February 29	
09:30–09:40	<i>Welcome & Opening</i>
09:40–10:30	Getting to know each other: Mathematical speed dating I
10:30–11:00	Coffee Break
11:00–12:00	Getting to know each other: Mathematical speed dating II
14:30–15:00	Kounchev: <i>An Almansi Type Formula on the Sphere and New Cubature Formulas with Error Bounds</i>
15:00–15:30	Nagler: <i>Lower Estimates for Smooth Data Approximation</i>
15:30–16:15	Coffee & Cake
16:15–16:45	Rabut: <i>Least square Bézier or B-spline curves</i>
16:45–17:15	Buhmann: <i>Multiquadrics Multivariate Interpolation without Added Constants</i>
Tuesday, March 1	
09:00–9:30	Forster: <i>Interpolation with Complex B-splines</i>
09:30–10:00	Massopust: <i>Fractional and Complex Pseudo-Splines</i>
10:30–11:00	Coffee Break
11:00–11:30	Christensen: <i>Wavelet Frames and the Unitary Extension Principle</i>
11:30–12:00	Zhuang: <i>Canonical Quincunx Tight Framelets</i>
14:30–15:00	Volontè: <i>Radial Kernels via Scale Derivatives and Wavelets</i>
15:00–15:30	Berstein: <i>Riesz Based Multiresolution Analysis in Image Processing</i>
15:30–16:15	Coffee & Cake
16:15–16:45	Gütschow: <i>Rotational Anisotropic Wavelet Transform</i>
16:45–17:35	Führ: <i>Continuous Wavelet Transforms in Higher Dimensions</i>
Wednesday, March 2	
09:00–09:30	Sissuono: <i>Numerical Inpainting Techniques – Modeling with Inherent Discretization</i>
09:30–10:20	Beckmann: <i>Error Estimates and Convergence Rates for Filtered Back Projection</i>
10:20–11:10	Filbir: <i>Detection of Hidden Frequencies: An OPUC Approach</i>
11:10–11:30	Coffee Break
11:30–12:00	Dyn: <i>Linear Pyramid Multiresolutions Based on Reverse Subdivision Schemes</i>
12:00–12:30	Yáñez: <i>Non-Consistent Cell-Average Multiresolution Operators: The Case of the PPH Nonlinear Prediction Operator</i>
	Excursion
Thursday, March 3	
09:00–09:30	Conti: <i>From Box-Splines to Four Directional Pseudo-Splines</i>
09:30–10:00	Novara: <i>Limit Stencils of Non-stationary Approximating Schemes and their Applications</i>
10:00–10:30	Sabin: <i>C² Subdivision Revisited</i>
10:30–11:00	Coffee Break
11:00–11:30	Romani: <i>Approximation order of non-stationary subdivision schemes</i>
11:30–12:00	Donat: <i>A Family of Non-Oscillatory, 6-point, Interpolatory Subdivision Schemes</i>
14:30–15:20	Moosmüller: <i>Hermite Subdivision on Manifolds</i>
15:20–16:15	Coffee & Cake
16:15–16:45	Cotronei: <i>Hermite Subdivision Preserving Exponentials: Factorization and Convergence Issues</i>
16:45–17:35	Turati: <i>Multigrid Methods and Subdivision Schemes</i>
Friday, March 4	
09:00–09:50	Maier: <i>Scattered Data Problems on Submanifolds: Ambient Solutions</i>
09:50–10:40	Diederichs: <i>Projection-based Parameter Estimation for Bivariate Exponential Sums</i>
10:40–11:10	Coffee Break
11:10–12:00	Sauer: <i>Prony's Method and Linear Algebra for Nonlinear Problems</i>

Error Estimates and Convergence Rates for Filtered Back Projection

Matthias Beckmann, Armin Iske*

Computerized tomography allows us to reconstruct a bivariate function from given Radon samples. The reconstruction is based on the *filtered back projection* (FBP) formula, which describes the analytical inversion of the Radon transform. The FBP formula, however, is highly sensitive with respect to noise and, thus, numerically unstable. To overcome this problem, suitable low-pass filters with a compactly supported window function and of finite bandwidth are employed.

The objective of this talk is to analyse the intrinsic FBP reconstruction error which is incurred by the use of a low-pass filter. To this end, we derive L^2 -error estimates for the relevant case of target functions from Sobolev spaces of fractional order. The obtained error bounds are affine-linear with respect to the distance of the filter's window function to the constant function 1 in the L^∞ -norm. Assuming more regularity of the window function, we can refine our error estimates to prove convergence for the FBP reconstruction in the L^2 -norm as the filter's bandwidth goes to infinity. Further, we determine asymptotic convergence rates in terms of the bandwidth of the low-pass filter and the smoothness of the target function.

Riesz Based Multiresolution Analysis in Image Processing

Swanhild Bernstein

The analytic signal is an important complex-valued representation in one-dimensional signal processing. It has a variety of applications for example in radar-based object detection, processing of seismic data, speech recognition, airfoil design.

One of the most important property of the analytic signal is the split of identity, i.e. a polar representation as a complex signal.

A higher dimensional analog that has most of the properties of the analytic signal is the monogenic signal, introduced by Felsberg and Sommer [1]. In 2D the monogenic signal is exactly the spiral phase quadrature formula found by Larkin et al. [2] in optical signal processing. The monogenic signal is based on Riesz transforms an quaternions or Clifford algebras. Based on the Riesz transforms a higher dimensional Riesz-Hilbert transform which builds the monogenic signal.

A signal is usually assumed to be band-limited, is mostly obtained by isotropic wavelets. Held et al. [3] combined these with the concept of monogenic signals and constructed monogenic wavelet frames. This scale-based approach provides monogenic information in a multiscale setup.

We will represent the idea of a monogenic signal and its polar form that gives a split of identity in amplitude, phase and local orientation.

A specific task in image processing is to highlight specific edges. That can be done by a fractional Riesz-Hilbert transform which we will introduce and explain.

Further, the multiresolution analysis of wavelet frames can be used for structural analysis. We will explain that also in the talk.

References

- [1] M. Felsberg, G. Sommer, *The monogenic signal*, IEEE Trans. Signal Proc. 49(12), 2001, 3136–3144.
- [2] K.G. Larkin, D.J. Bone, M.A. Oldfield, *Natural demodulation of two-dimensional fringe pattern. I. General background of the spiral phase quadrature transform*, J. Opt. Soc. Am. A 18(8) (2001), 1862–1870.
- [3] S. Held, M. Storath, P. Massopust, B. Forster, *Steerable wavelet frames based on the Riesz transform*, IEEE Trans. on Image Proc., 19(3) (2010), 653–667.

Multiquadrics Multivariate Interpolation without Added Constants

Martin Buhmann

The radial basis function approach is a highly useful method to approximate multivariate functions especially when kernel methods are required as generalised finite elements and in particular when spatial dimensions are high. There are several reasons for this, including their flexibility with respect to the required smoothness, the existence of interpolants, the independence of dimension, the accuracy of ensuing results. Among the radial basis functions used for approximation and interpolation when the solutions of partial differential equations are sought, the so-called multiquadric function turned out to be useful as it is flexible and provides accurate approximations. While it was noted by R. Hardy and proved in a famous paper by C. A. Micchelli that radial basis function interpolants exist uniquely for the multiquadric radial basis function $\varphi(r)$ as soon as least two centres are pairwise distinct, the error bounds for this interpolation problem always demanded an added constant to s . By using Pontryagin native spaces, we obtain error bounds that no longer require this additional constant expression. (Paper with O. Davydov)

Wavelet Frames and the Unitary Extension Principle

Ole Christensen

Frames is a functional analytic tool that allows to obtain decompositions of elements in a Hilbert space, in a fashion that is similar to the ones obtained via orthonormal bases but with much more flexibility. We will review the theory for frame constructions of the wavelet type, obtained via the unitary extension principle by Ron & Shen and its variants. Based on recent joint work with Forster and Massopust new frame constructions will be presented.

From Box-Splines to Four Directional Pseudo-Splines

Costanza Conti

Univariate pseudo-splines are a generalization of uniform B-splines and interpolatory $2n$ -point subdivision schemes. Each pseudo-spline is characterized as the subdivision scheme with least possible support among all schemes with specific degrees of polynomial generation and reproduction. In this talk, after describing univariate pseudo-splines we consider the problem of constructing the symbols of the bivariate counterpart on the four-directional grid of the plane. In particular, we provide a formula for the symbols of a family of symmetric four-directional bivariate pseudo-splines. All methods employed in this work are of purely algebraic nature.

This is a joint work with C. Deng and K. Hormann

Hermite Subdivision Preserving Exponentials: Factorization and Convergence Issues

Mariantonia Cotronei

The talk deals with *level-dependent* Hermite subdivision schemes preserving elements in the space spanned by $\{1, x, \dots, x^p, e^{\pm\lambda_1 x}, \dots, e^{\pm\lambda_r x}\}$. We illustrate how such preservation property allows for a factorization of the subdivision operators at each level in terms of the so called *annihilator operator*. We further present some conditions for the convergence of the scheme which make use of such factorization.

This is a joint work with Costanza Conti and Tomas Sauer.

Projection-based Parameter Estimation for Bivariate Exponential Sums

Benedikt Diederichs

Estimating the spectrum of a signal is a key problem in many fields of application. While the case of univariate signals is classic, the multivariate case has attracted a lot of interest in recent years. We tackle the multivariate problem by splitting it into several univariate problems. This is done by sampling along multiple lines. Depending on whether one chooses parallel or non-parallel lines, different strategies are necessary. In this talk we discuss these different approaches, give recover guarantees and provide some numerical examples.

A Family of Non-Oscillatory, 6-point, Interpolatory Subdivision Schemes

Rosa Donat

We propose and analyze a new family of nonlinear subdivision schemes based on a weighted analog of the harmonic mean. Due to their design, these schemes can be considered as nonlinear versions of the 6-point Deslauriers-Dubuc interpolatory scheme, just as the Power_p schemes are nonlinear, non-oscillatory versions of the 4-point Deslauriers-Dubuc interpolatory scheme.

Linear Pyramid Multiresolutions Based on Reverse Subdivision Schemes

Nira Dyn

The talk will start with a review of linear pyramid multiresolutions based on interpolatory subdivision schemes, and of their properties, such as stability and decay rates of the generated details. We will then present our approach for extending these pyramid multiresolutions to non-interpolatory subdivision schemes, which requires to "reverse" these schemes. We will give a sufficient condition for the existence of a reverse scheme of our type, and will give examples of known schemes satisfying this condition. Results on the stability of these multiresolutions and the decay rates of the generated details will be presented.

This talk is based on a joint work with Xiaosheng Zhuang.

Detection of Hidden Frequencies: An OPUC Approach

Frank Filbir

Many applications in digital signal processing, speech recognition, as well as applications in numerical analysis lead to the question of recuperating the location of the support of an atomic measure from its moments. In case the underlying space is the torus this problem is often called the problem of hidden frequencies and it is related to super-resolution of signals. In this talk we present an approach based on orthogonal polynomials. We will demonstrate the effectiveness and robustness of the method.

Interpolation with Complex B-splines

Brigitte Forster

Cardinal B-splines of complex order s , for short, complex B-splines, are natural extensions of classical Curry-Schoenberg (polynomial) B-splines B_n , where the order $n \in \mathbb{N}$ is replaced by a complex number s . These complex B-splines inherit many of the important and interesting properties of the B_n . In this talk, we will concentrate on the interpolation property. Whereas for the fractional B-splines B_α , $\alpha > 1$, the interpolation property can be easily verified, this is not obvious for the complex case. In fact, this question is closely related to the growth conditions and the distribution of zeros of sums of Hurwitz zeta functions. In the talk, we give a positive answer to the question of interpolation with complex B-splines for a certain range of complex degrees. The complete characterization of the admissible degrees is still an open question.

This is joint work with Ramūnas Garunkštis (Vilnius University, Lithuania), Peter Masopust (Technische Universität München) and Jörn Steuding (Würzburg University).

Continuous Wavelet Transforms in Higher Dimensions

Hartmut Führ

The generalized continuous wavelet transforms that this talk is about are constructed by choosing a suitable matrix group, the so-called dilation group H . Wavelet systems associated to this group then arise by picking a suitable wavelet, dilating it by elements of H , and translating arbitrarily. The wavelet transform of a signal (a function or tempered distribution) then arises by taking scalar product with the elements of the wavelet system.

In higher dimensions, there is a large variety of suitable matrix groups to choose from. One of the pertinent questions in connection with the associated transforms is whether they are able to efficiently encode important properties of the analyzed functions, specifically smoothness behaviour. So far, these questions have only been studied for a few isolated groups.

For example, it is well-known that the homogeneous Besov spaces in any dimension are related to the continuous wavelet transform associated to the similitude group. Another, celebrated, example is the shearlet group and its associated systems of wavelets, called shearlets. Shearlet coorbit space norms, obtained by imposing weighted mixed L^p norms on the shearlet coefficients, can be understood as a quantification of (directional) global smoothness, whereas local directional smoothness (or roughness) features such as the wavefront set are captured by local decay behaviour of the shearlet coefficients.

In this talk I will give an overview of recently developed methods that allow to study the above-mentioned properties of wavelet systems in a unified and comprehensive framework, for a large variety of dilation groups. The new techniques provide far-reaching extensions of the above-mentioned results for shearlets.

Rotational Anisotropic Wavelet Transform

Silja Gütschow, Tomas Sauer*

It is known that the shearlet transform can evaluate the wavefront set (see [1] and [2]). Nevertheless, in applications shearing can be quite troublesome, because the picture will get very wide and flat. Therefore, another transform, the rotational anisotropic wavelet transform has to be used and will be presented here, which works with rotations and anisotropic scaling matrices. It will be shown that the representation is meaningful from representation theoretic point of view. Furthermore, this transform can evaluate the wavefront set of line distributions while keeping the size of the image constant.

References

- [1] G. Kutyniok and D. Labate. Resolution of the wavefront set using continuous shearlets. *Transactions of the American Mathematical Society*, 361:2719- 2754, 2009.
- [2] P. Grohs. Shearlets and microlocal analysis. In: *Shearlets: Multiscale Analysis for Multivariate Data*, Birkhäuser-Springer, 2012.

An Almansi Type Formula on the Sphere and New Cubature Formulas with Error Bounds

Ognyan Kounchev

We find a representation of all harmonic polynomials in a form which is naturally termed Almansi type formula on the sphere. This provides a convenient approach to the definition of new cubature formulas for the computation of integrals of functions having limited smoothness (or even singularities) on the sphere.

The advantage of these new formulas is the error bound which is not available for the other approaches.

Scattered Data Problems on Submanifolds: Ambient Solutions

Lars-Benjamin Maier

The talk will cover novel approaches to scattered data problems on closed submanifolds, i.e. approximation problems with only a set of discrete data sites Ξ from a compact submanifold $\mathbb{M} \subseteq \mathbb{R}^d$ without boundary and corresponding function values Y given. It will present solutions that are obtained by suitable functional minimization methods for function spaces in an ambient neighbourhood of \mathbb{M} , thus avoiding difficulties that arise in common approaches.

A special focus will be set on interpolation problems for sparse interpolation sites and smooth extrapolation from these sparse sites into regions of \mathbb{M} without any data sites. Furthermore, the interpolation concept will be extended into a more general concept of smoothing over \mathbb{M} and in the data sites.

Fractional and Complex Pseudo-Splines

Peter Massopust

Pseudo-splines of integer order (m, ℓ) were introduced by Daubechies, Han, Ron, and Shen as a family which allows interpolation between the classical B-splines and the Daubechies' scaling functions. The purpose of this paper is to generalize the pseudo-splines to fractional and complex orders (z, ℓ) with $\alpha := \operatorname{Re} z \geq 1$. This allows increased flexibility in regards to smoothness: instead of working with a discrete family of functions from C^m , $m \in \mathbb{N}$, ones uses a continuous family of functions belonging to the Hölder spaces C^α , $\alpha \geq 1$. The presence of the imaginary part of z allows for direct utilization in complex transform techniques for signal and image analyses.

Hermite Subdivision on Manifolds

Caroline Moosmüller

Hermite subdivision schemes are refinement algorithms that operate on discrete point-vector data and produce a curve and its derivatives in the limit. Most results on Hermite schemes concern data in vector spaces and rules which are linear. We are interested in Hermite schemes that operate on manifold-valued data and are defined by the intrinsic geometry of the underlying manifold (in particular, by geodesics and parallel transport). In this talk we adapt a linear Hermite scheme to the manifold setting and analyse the resulting nonlinear scheme with respect to convergence and C^1 smoothness. For this purpose we use the method of proximity, which enables us to conclude smoothness properties of a manifold-valued scheme from the linear scheme it is derived from.

Lower Estimates for Smooth Data Approximation

Johannes Nagler

The aim of this talk is to show a general functional analysis based framework to prove lower estimates for the approximation error of operators with smooth range by K -functionals using spectral properties.

There are mainly two conditions on an operator T to derive lower estimates. The proposed technique uses the convergence of the iterates of the operator. If in addition the limiting operator is annihilated by a differential operator, then lower estimates can be derived. We will show first that the iterates of every positive linear operator of finite-rank having a partition of unity property converges by spectral properties. Then we will generalize our setting to so called quasi-compact operators, where all limiting points in the spectrum are inside the unit ball. This setting guarantees that the fixed point space of the operator and its adjoint is in fact finite-dimensional. Provided that bases for these spaces are known, we show how to obtain the limiting operator of the iterates. This result is constructive in the sense that the limit operator can be explicitly calculated by the inverse of a Grammian matrix.

Limit Stencils of Non-Stationary Approximating Schemes and their Applications

Paola Novara, Lucia Romani*

To construct smooth surfaces interpolating the vertices of meshes with arbitrary topology we could use one of the several interpolatory subdivision schemes proposed in literature. However, with this kind of schemes we do not have a lot of flexibility in the shape of the limit surface and its quality is very often not satisfactory due to undesired undulations that frequently appear. To overcome these deficiencies in the limit surface, we present a new efficient method for constructing subdivision surfaces that interpolate a given quadrilateral mesh with arbitrary topology by means of the limit stencil of a non-stationary approximating scheme. This method, together with the use of non-stationary rules, let us gain two free parameters that could be set to suitable adjust the shape of the desired limit surface. To illustrate our strategy, we here compute the limit stencil of non-stationary Doo-Sabin's scheme and show the variety of C^1 interpolating surfaces that can be easily obtained. Moreover, we point out our ongoing research on the derivation of the limit stencil of non-stationary Catmull-Clark's scheme with the final aim of producing smoother interpolating surfaces (C^2 almost everywhere) with the same freedom of shape

Least square Bézier or B-spline curves

Christophe Rabut

Given a control polygon, we use some least-square criterion, to derive “B-curves” (i.e. Bernstein, B-spline, hyperbolic or circular spline curves...) which are closer the control polygon, still being in the same vectorial space as the original one. We usually loose the convex hull property, and better preserve the general shape of the control polygon. The idea is simple : we minimize some L^2 -distance between the curve and the control polygon.

Furthermore, by increasing (resp. decreasing) the degree of the parametric polynomial curve, in the same way we derive a curve still closer (resp. further) the control polygon. Similarly we obtain the same type of results by increasing (resp. decreasing) the number of knots of the spline curve

Actually the so-obtained curves (or surfaces, or any multi-dimensional object) are in the space generated by the original B-functions, and some “new control points” are easily derived. The obtained curve (or surface) is the quasi-interpolation, by using the original B-functions, of the so-obtained “new control polygon”. We so keep all the known properties of the original quasi-interpolation (including convex hull property), express in this “new control polygon”.

We give ways to derive new B-functions which are linear combinations of the original B-functions (i.e. Bernstein polynomials, B-splines, hyperbolic or circular B-spline curves...), such that the associated Bézier curve is closer the control polygon than the usual one (still being in the same functional space, but possibly not in the convex hull of the control polygon), and better preserving the general shape of the control polygon. We also give ways to derive associated basis functions such that the so-obtained curve is further form the control polygon (more cutting the angles), while being in the same functional space.

Finally we propose to mix this least-square criterion together with a least-square distance between some points of the curve and the control points, and with a variational criterion aiming to cut down the oscillations of the curve.

The same strategy is possible for surfaces by using corresponding B-surfaces, such as tensorial product of Bernstein polynomials, of B-splines, or by using polyharmonic B-splines.

Approximation Order of Non-Stationary Subdivision Schemes

Lucia Romani

The main goal of this talk is to investigate the approximation order of non-stationary subdivision schemes. More precisely, we provide non-stationary counterparts of the well-known result asserting that the reproduction of an N -dimensional space of polynomials is sufficient for a convergent stationary subdivision scheme to have approximation order N .

This work is in collaboration with Costanza Conti (University of Florence, Italy), Paola Novara (University of Insubria - Como, Italy) and Jungho Yoon (Ewha Womans University - Seoul, South Korea).

C^2 Subdivision Revisited

Malcolm Sabin

An outstanding issue with the industrial acceptance of subdivision surfaces is the question of continuity at extraordinary vertices.

The talk gives a little background of the known theory, and then provides a hunch that for a subdivision scheme to have quadratic generation near EVs it must either have subdominant eigenvalues at the EV of $1/2$, or else have a stable quasi-interpolation degree of at least 2.

A sounder result follows, that the quasi-interpolation condition places a lower bound on the number of entries in each stencil.

These ideas are currently being used to try to construct a variant of the butterfly scheme C^2 everywhere.

Prony's Method and Linear Algebra for Nonlinear Problems

Tomas Sauer

Prony's method reconstructs the frequencies and coefficients of a function of the form

$$f(x) = \sum_{\omega \in \Omega} f_{\omega} e^{\omega \cdot x}, \quad f_{\omega} \in \mathbb{C} \setminus \{0\}, \quad \omega \in \Omega \subset \mathbb{R}^s + i(\mathbb{R}/2i\pi\mathbb{Z})^s$$

as long as Ω is a *finite* set. The set e^{Ω} is determined implicitly as the common zeros of a finite set of polynomials, hence as the solution of a system of polynomials equations. By means of proper algebraic methods, this system can be set up and solved entirely by techniques from Numerical Linear Algebra which will be described in the talk. In particular, this leads to very fast and quite stable algorithms for the solution of Prony's problem.

Numerical Inpainting Techniques – Modeling with Inherent Discretization

Nada Sissouno

Inpainting methods are used for the reconstruction of parts of an image that are damaged or missing. Those methods strongly depend on the size and shape of those parts, and also on the structure and texture of the image. In this talk we will focus on structured images and thin inpainting areas, where PDE-based and variational inpainting approaches have proven to be very useful. We will present an inpainting method which is efficiently computable and automatically provides a discretization of those continuous inpainting models. This is joint work with Tomas Sauer.

Multigrid Methods and Subdivision Schemes

Valentina Turati

Multigrid methods are well-known to be optimal methods for a variety of problems including, but not limited to the solution of partial differential equations. Subdivision schemes are efficient algorithms for curve and surface design, with important applications in computer graphics and computer aided geometric design. The purpose of this talk is to provide an overview of multigrid methods focusing on their connections with subdivision schemes. In particular, we show how the properties of polynomial generation and L^∞ -stability of integer translates of the basic limit function of a convergent subdivision scheme can be used to define new grid transfer operators capable of reducing the number of iterations or the computational cost of multigrid methods.

This work is in collaboration with Maria Charina (University of Vienna), Marco Donatelli (University of Insubria - Como) and Lucia Romani (University of Milano-Bicocca).

Radial Kernels via Scale Derivatives and Wavelets

Elena Volontè

In this talk I present how it is possible to generate various new radial kernels by taking derivatives of known kernels with respect to a scale parameter and to provide a simple recipe that explicitly constructs new kernels from the negative Laplacian of known ones. Surprisingly, these two methods are proven to coincide for certain standard classes of radial kernels. I will give you some examples of new kernels and discuss their additional properties that make them attractive. This construction is interesting because the new kernels are wavelets satisfying the admissibility condition. So we have a dictionary of generators of $L^2(\mathbb{R}^d)$ that are radial wavelets. Moreover it is possible to consider from them discrete wavelet generators of $L^2(\mathbb{R}^d)$.

Non-Consistent Cell-Average Multiresolution Operators: The Case of the PPH Nonlinear Prediction Operator

*Rosa Donat, Dionisio Yáñez**

A general framework about multiresolution representation has been presented by Harten in [5]. The Harten's schemes are based on two operators: decimation, D , and prediction, P , that satisfy the consistency property $DP = I$, where I is the identity operator. In this work we introduce and analyze a new non-linear subdivision scheme based on the definition of a weighted difference. In order to establish this concept we follow the ideas showed by Marquina and Serna in [6]. We study the convergence and the stability of the schemes. These operators do not satisfy the consistency property. In order to apply them in a Harten's multiresolution scheme we use the algorithm developed by Aràndiga and Yáñez in [3]. We analyze the applicability of these schemes to compress and remove noise to digital signals and images. We compare our method with the operators obtained using classical techniques (see [2,4]) and non-linear PPH function (see [1]). Some numerical tests are showed.

References

- [1] Amat, S.; Dadourian, K.; Liandrat, J.; Ruiz, J.; Trillo, J.C.: On a class of L^1 -stable nonlinear cell-average multiresolution schemes, *Journal of Computational and Applied Mathematics* 234 (2010) 1129–1139.
- [2] Aràndiga, F.; Donat, R.: Nonlinear multiscale decompositions: The approach of A. Harten, *Numerical Algorithms* 23 (2000) 175–216.
- [3] Aràndiga, F.; Yáñez, D. F.: Non-consistent cell-average multiresolution operators with application to image processing, *Applied Mathematics and computation* 272-1 (2016), 208–222.
- [4] Cohen, A.; Daubechies, I.; Feauveau, J.: Biorthogonal bases of compactly supported wavelets, *Comm. Pure Appl. Math.* 45 (1992) 485–560.
- [5] Harten, A.: Multiresolution representation of data: General framework, *SIAM J. Numer. Anal.* 33 (1996) 1205–1256.
- [6] Serna, S. ; Marquina A.: Power ENO methods: a fifth-order accurate weighted power ENO method, *J. Comput. Phys.* 194-2 (2004) 632–658.

Canonical Quincunx Tight Framelet

Xiaosheng Zhuang

In this talk, we shall focus on the construction a family of quincunx tight framelets having the following desirable properties:

1. The high-pass filters have high order of vanishing moments.
2. The low-pass filter and all the high-pass filters possess symmetry.
3. The number of high-pass filters should be relatively small for computational efficiency.
4. The high-pass filters should have shortest possible support, more precisely, the supports of all high-pass filters should not be larger than the support of the low-pass filter.
5. The tight framelet filter bank has the canonical property.

We shall construct a family of symmetric canonical quincunx tight framelets that has the minimum number of generators (associated with double canonical quincunx tight framelet filter banks) and has increasing orders of vanishing moments. Family of multiple canonical quincunx tight framelets with symmetry and increasing order of vanishing moments can be also constructed using tensor product of one-dimensional filters.

List of participants

Matthias Beckmann, University of Hamburg, matthias.beckmann@uni-hamburg.de
Swanhild Bernstein, TU Freiberg, bernstei@math.tu-freiberg.de
Martin Buhmann, University of Giessen, Martin.Buhmann@math.uni-giessen.de
Ole Christensen, Technical University of Denmark, ochr@dtu.dk
Costanza Conti, University of Florence, costanza.conti@unifi.it
Mariantonia Cotronei, University of Reggio Calabria, mariantonia.cotronei@unirc.it
Benedikt Diederichs, University of Hamburg, benedikt.diederichs@uni-hamburg.de
Rosa Donat, University of Valencia, Rosa.M.Donat@uv.es
Nira Dyn, Tel Aviv University, niradyn@post.tau.ac.il
Frank Filbir, Helmholtz Center Munich, filbir@helmholtz-muenchen.de
Brigitte Forster-Heinlein, University of Passau, Brigitte.Forster-Heinlein@uni-passau.de
Hartmut Führ, RWTH Aachen, fuehr@MathA.rwth-aachen.de
Silja Gütschow, University of Passau, guetschow@forwiss.uni-passau.de
Svenja Hüning, TU Graz, Svenja.Huening@t-online.de
Ognyan Kounchev, , okounchev@gmail.com
Lars-Benjamin Maier, TU Darmstadt, maier@mathematik.tu-darmstadt.de
Peter Massopust, TU Munich, massopust@ma.tum.de
Caroline Moosmüller, TU Graz, moosmueller@tugraz.at
Johannes Nagler, University of Passau, johannes.nagler@uni-passau.de
Paola Novara, University of Insubria, paola.novara@uninsubria.it
Christophe Rabut, INSA Toulouse, christophe.rabut@insa-toulouse.fr
Ulrich Reif, TU Darmstadt, reif@mathematik.tu-darmstadt.de
Lucia Romani, University of Milano-Bicocca, lucia.romani@unimib.it
Malcolm Sabin, Numerical Geometry Ltd., malcolm.sabin@btinternet.com
Tomas Sauer, University of Passau, Tomas.Sauer@uni-passau.de
Nada Sissouno, University of Passau, nada.sissouno@uni-passau.de
Valentina Turati, University of Wien, turativ35@univie.ac.at
Elena Volontè, University of Milano-Bicocca, e.volonte2@campus.unimib.it
Dionisio Yanez, University of Valencia, dionisio.yanez@uv.es
Xiaosheng Zhuang, City University of Hong Kong, xzhuang7@cityu.edu.hk