

Sampling and the Energy Concentration Problem: A New Perspective

Ahmed I. Zayed
DePaul University
Chicago, IL, U. S. A

Abstract

The Whittaker-Shannon-Kotel'nikov (WSK) sampling theorem plays an important role in communication engineering because it enables engineers to reconstruct bandlimited signals from their samples at a discrete set of points.

Recently, G. Walter and X. Shen introduced another sampling theorem for bandlimited signals in one variable using the prolate spheroidal wave functions (PSWFs), which are eigenfunctions of the integral operator

$$\mathcal{F}(\varphi)(x) = \int_{-1}^1 \varphi(t) \frac{\sin(c(x-t))}{\pi(x-t)} dt. \quad (1)$$

and the differential operator L_c ,

$$L_c\varphi(x) = (1-x^2) \frac{d^2}{dx^2} \varphi(x) - 2x \frac{d}{dx} \varphi(x) - c^2 x^2 \varphi(x), \quad c > 0, \quad (2)$$

They also appear in the solution of the energy concentration problem, which is the problem of finding a bandlimited signal with maximum energy concentration in the interval $(-T, T)$ in the time domain.

In this talk we present a new perspective on sampling and the energy concentration problem by introducing a new generalization of PSWFs.