Construction of smooth windows generating dual pairs of Gabor - and wavelet frames

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Gabor - and wavelet frames are mathematical tools which allow a convenient way of obtaining series expansions of functions in $L^2(\mathbb{R})$. In order to do this in pratice it is necessary to be able to explicitly construct dual frame pairs. This talk will be concerned with a new method to explicitly construct dual pairs of Gabor - and wavelet frames. In addition we discuss the convenient constructions which it gives rise to.

Abstract

Let g be any real-valued, bounded and compactly supported function, whose integer-translates $\{T_k g\}_{k\in\mathbb{Z}}$ form a partition of unity. Based on a new construction of dual windows associated with Gabor frames generated by g, we present a method to explicitly construct dual pairs of Gabor frames. This new method of construction is based on a family of polynomials which is closely related to the *Daubechies polynomials*, used in the construction of compactly supported wavelets. For any $k \in \mathbb{N} \cup \{\infty\}$ we consider the socalled *Meyer scaling functions* and use these to construct compactly supported windows $g \in C^k(\mathbb{R})$ associated with a family of smooth compactly supported dual windows $\{h_n\}_{n=1}^{\infty}$. For any $n \in \mathbb{N}$ the pair of dual windows $g, h_n \in C^k(\mathbb{R})$ have compact support in the interval $I = [-2/3, 2/3]$ and share the property of being constant on half the length of their support. We therefore obtain arbitrary smoothness of the dual pair of windows g, h_n without increasing their support. We carry over the new construction of dual windows to the setting of wavelet frames. As a special case of our results we construct a joint dual window $\tilde{\psi} \in C^\infty(\mathbb{R})$ for a class of windows generating wavelet frames.

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