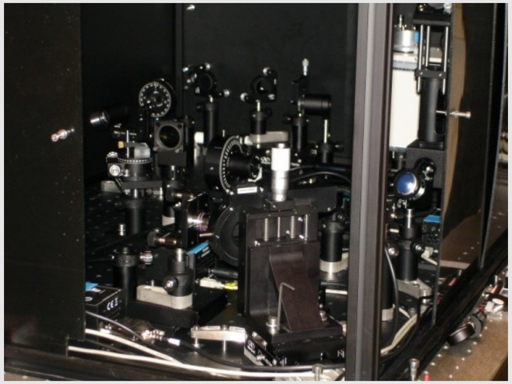
Fourier plane filtering, Riesz transform, and singularities in optics

Bettina Heise

Johannes Kepler University Linz
CDL MS-MACH,
Austria

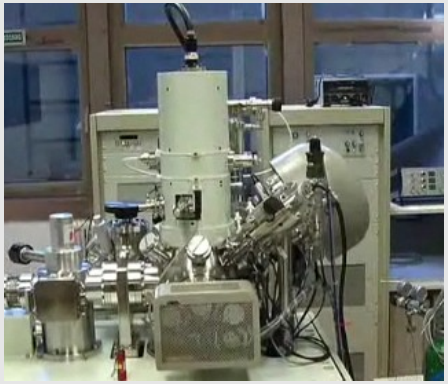
Content

- Physicist at CDL MS-MACH (Physics and Mathematics Department)
- CDL MS-MACH: Christian Doppler Laboratory for microscopic and spectroscopic material characterization
- Optical imaging (microscopy and interferometry) & signal- and image processing



Probing of:

-Metals, Coated metals,
-Polymers



Material research:

Optical characterization

Material research:

Spectroscopic characterization

Content

- Fourier plane filter, Riesz transform, Singularities,..
- Optics and Optical realizations (in microscopy and interferometry)
- Image processing

Image Processing:

Image Processing

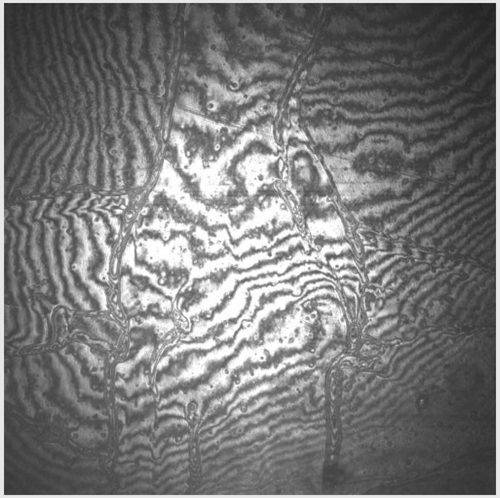


Image Processing | Natural scene images

Image Processing

Edges or corners:

•Gradient operations (GR)

•Phase congruency (→Kovesi,...) (PC)

•Wavelet or shearlet based detection (→Labate, →Kutyniok, → Steidl,...) (WM)

•Riesz kernel based (→ Felsberg, → Unser,...) (RLK)

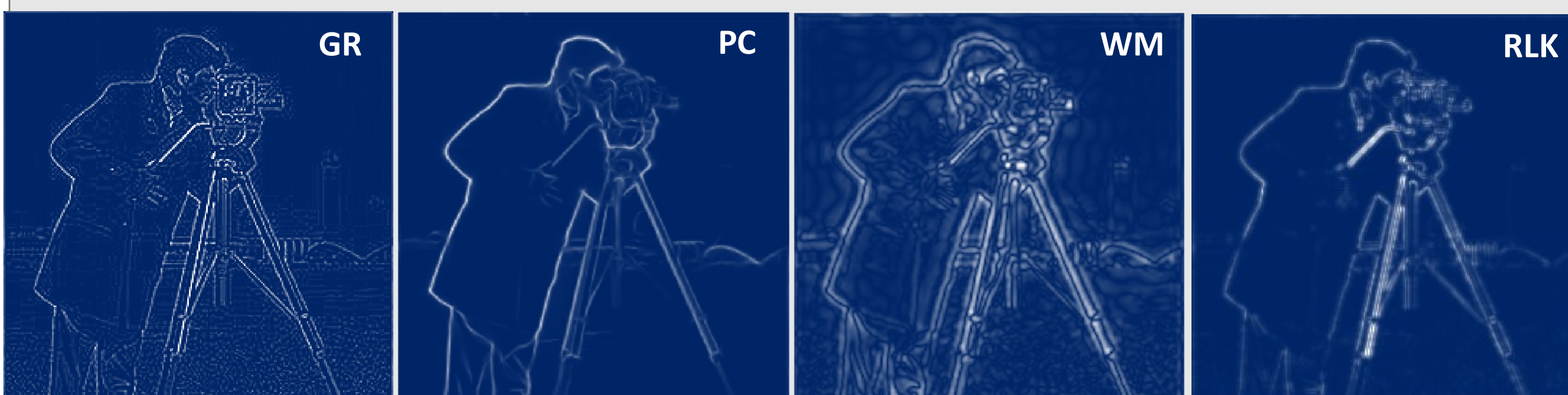
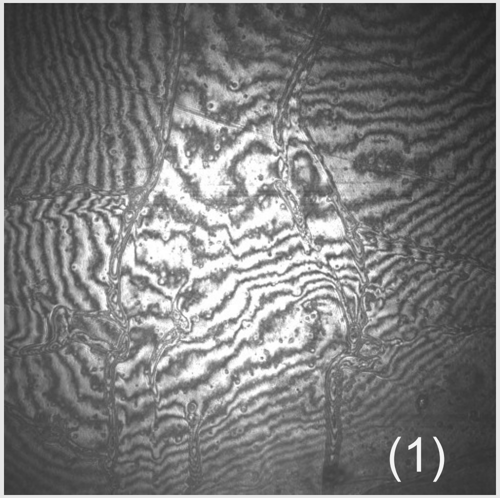


Image Processing | Fringe pattern images

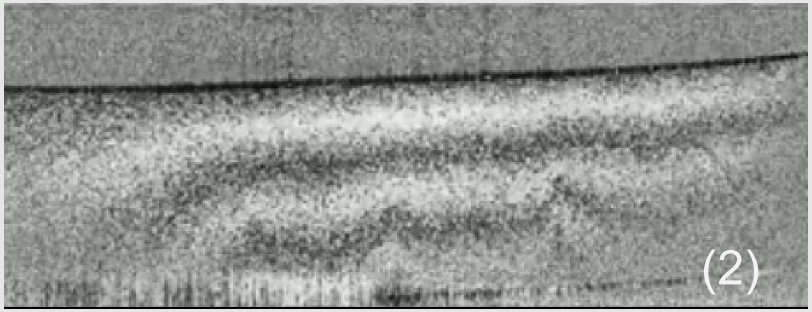
Image Processing



Fringe Pattern

- Interferometry (1),
- Polarization Sensitive -OCM imaging (2),
- Photoelasticity (3),
- ...Tree years ring

- Amplitude or frequency modulation, orientation



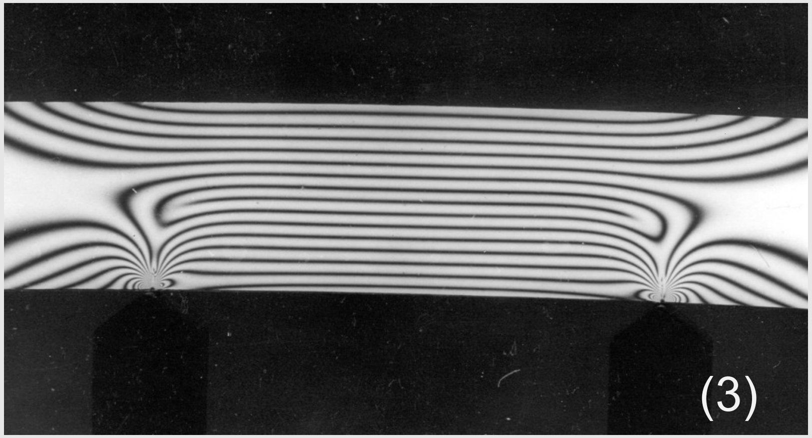
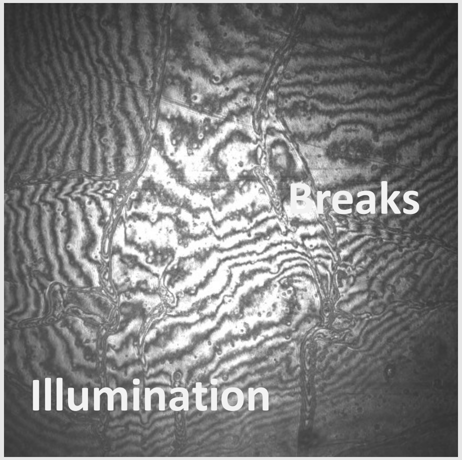
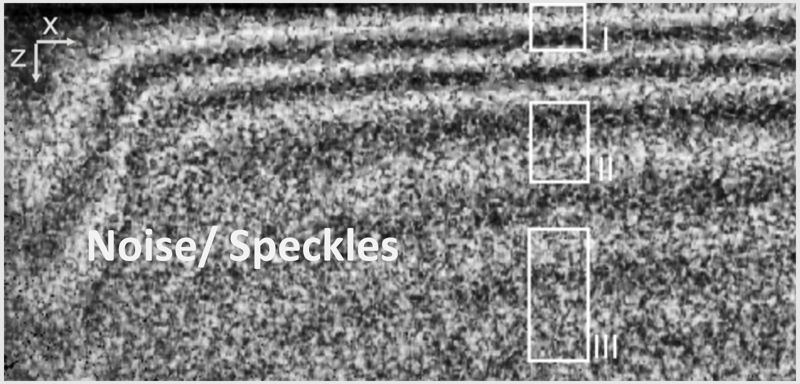
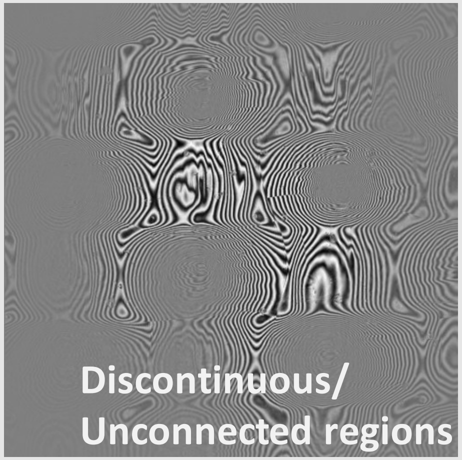


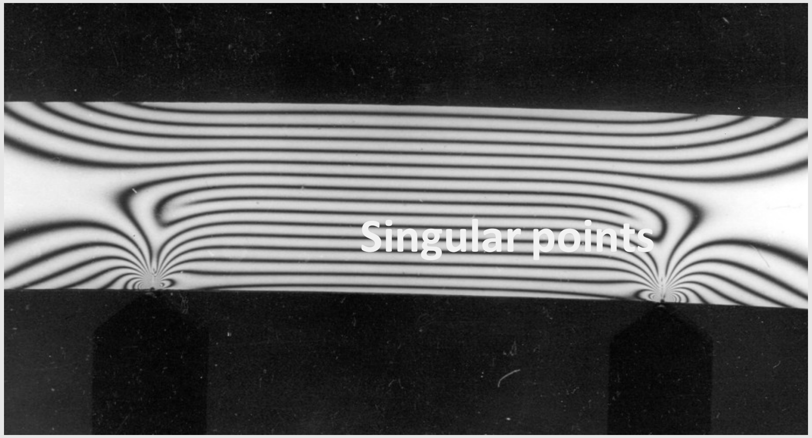
Image Processing | Artefacts

Image Processing





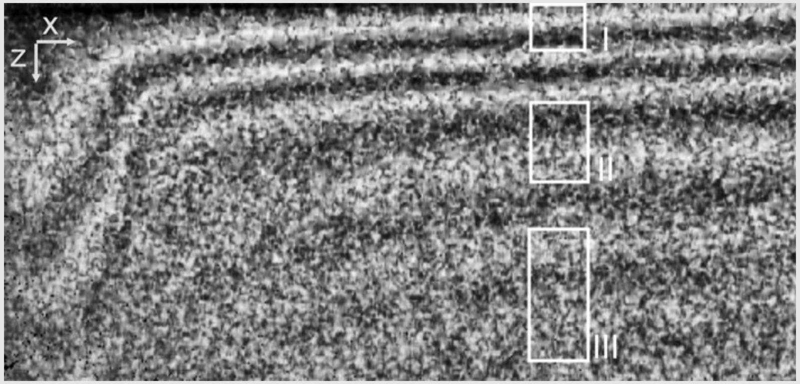




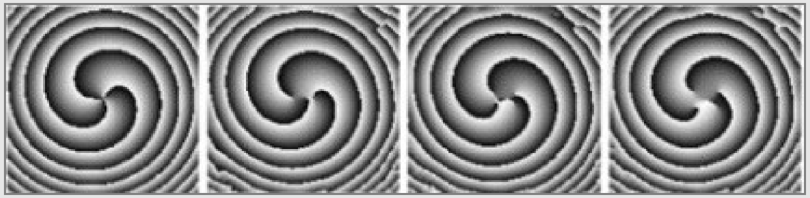
Imaging

Imaging

- Birefringence
- Scattering
- Degree of polarization uniformity



- Helicity / Topological charge
- Vortex orientation



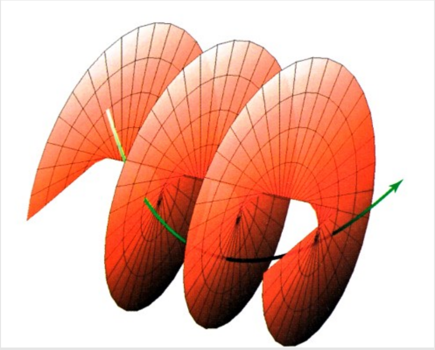
Imaging

Imaging

Imaging I Vortex / singular beam

Singularities in Optics

- Light as electro-magnetic wave field: amplitude and phase, polarization
    - plane , circular, helical wave fronts
    - Bessel beams
    - vortex/singular beams-> twisted light

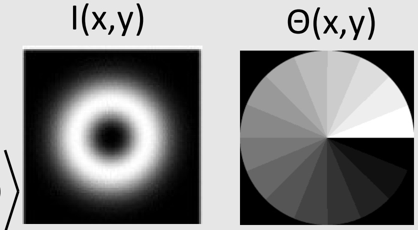


- Singular points: Phase is undefined, when amplitude is zero
- Optical vortices are characterized by carrying an orbital angular momentum and a phase that increases azimuthally about a singularity at the center of the beam
- Optical vortex:
    - wave field u(x,y)
    - Intensity I(x,y): "Doughnut"
    - az.Phase Θ(x,y):"Vortex/Spiral"

$$u(x,y) \propto \frac{1}{r^2} \exp[i\ l\ \theta]$$

$$I(x,y) \propto \left\langle u(x,y)\, u^*(x,y) \right\rangle$$

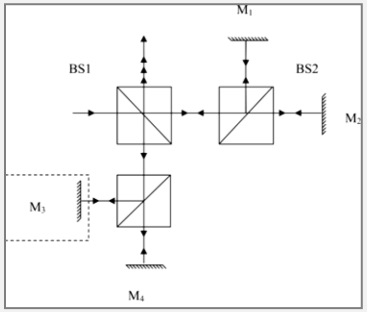
$$\theta(x,y) = angle(x+iy)$$

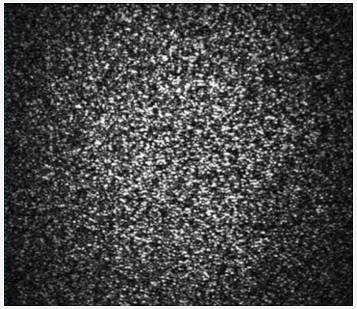


Imaging I Vortex beam generation

Vortex beam generations:

- Interference (3 beam interference)
- Random interferences at rough surfaces (speckle fields)
- Phase plates with helical profile





Imaging I Singular/ Vortex beams

Vortex beam applications:

-optical trapping and manipulation of micro-particles (tweezers)

-design of meta-materials (helical lattices)

-vortex interferometry

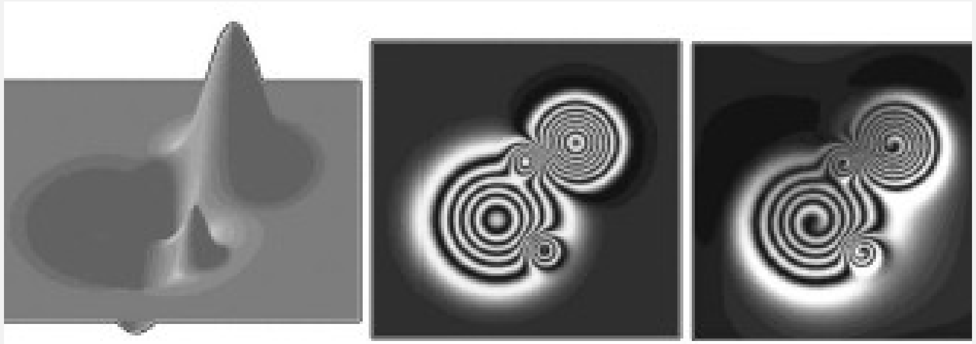
- spiral phase microscopy / spiral interferometry

- Fourier plane filtering

-stimulated emission depletion (STED) microscopy

-the encoding of optical quantum information

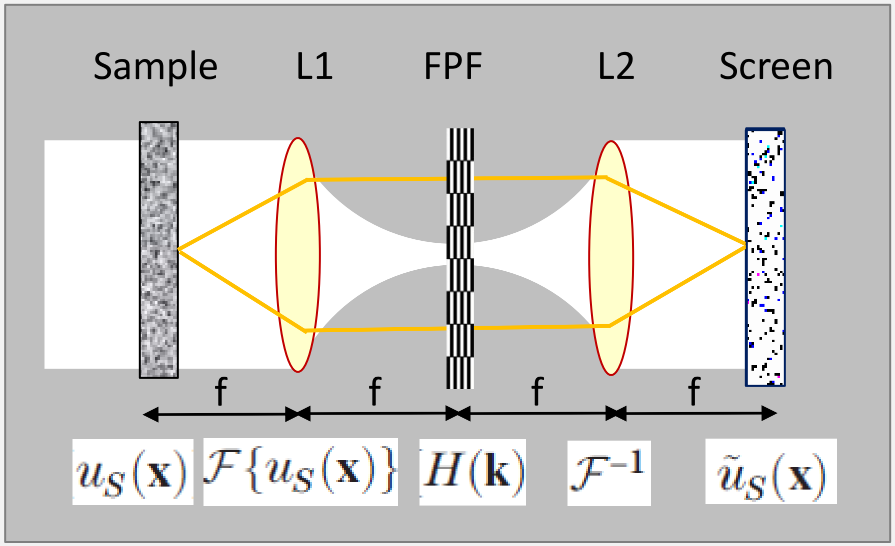
-...



- Fürhapter S, **Spiral interferometry,** Opt. Lett., 30(15), 2005.

Imaging I Fourier Plane Filtering Principle

Optical Fourier plane filtering represents linear system

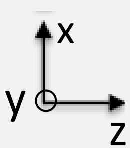


Spatial Domain — Fourier Domain

$\mathbf{x} = (x, y)$ $\qquad$ $\mathbf{k} = (k_x, k_y)$

Electric field

$$\tilde{u}_S(\mathbf{x}) \propto \mathcal{F}^{-1}\{H(\mathbf{k})\mathcal{F}\{u_S(\mathbf{x})\}\}$$



Principle of Fourier plane filtering

Mathematical description

Filter Function: $H(\mathbf{k}) = H_A(\mathbf{k})\, H_\Phi(\mathbf{k})$

Imaging I Fourier Plane Filtering in Microscopy

Fourier plane filtering history in microscopic imaging:

- Dark field microscopy (DF) (Amplitude, DC) $\langle I \rangle_{DC} = 0$
- Phase contrast microscopy (PC) (small phase) $I(x,y) \propto \varphi(x,y)$

  Zernike, (1930)
- Differential interference contrast (DIC) microscopy $I(x,y) \propto \frac{\partial \varphi(x,y)}{\partial x}$

  Nomarski, (1950)
- Schlieren imaging

  Foucault's knife edge test (1859)

  Thermal flow and pressure fields

$$I(x,y) \approx \frac{\partial \varphi(x,y)}{\partial x} \propto \frac{\partial n(x,y)}{\partial x}$$

$$I(x,y) \approx H_x\{\varphi(x,y)\} \approx H_x\{n(x,y)\}$$

- Spiral phase contrast microscopy

  M. Ritsch Marte, (2005)

$$I(x,y) \approx \left| A(x,y) + iR_x\{A(x,y)\} + jR_y\{A(x,y)\} \right|$$

$$I(x,y) \approx \left| \varphi(x,y) + iR_x\{\varphi(x,y)\} + jR_y\{\varphi(x,y)\} \right|$$

I(x,y): Measured intensity; A(x,y): Object wave amplitude; φ(x,y) : Object wave phase; n(x,y) refractive index, H_x: Partial Hilbert transform, R_x, R_y: Riesz transform components

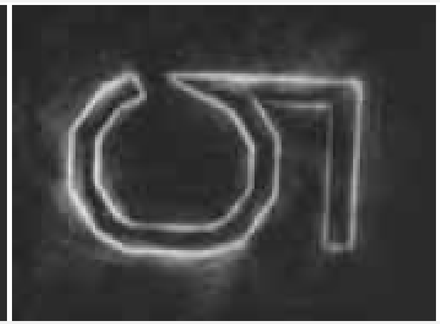
Imaging I Fourier Plane Filtering in Microscopy

Fourier plane filtering examples in microscopic imaging:

Amplitude objects

BF Image*

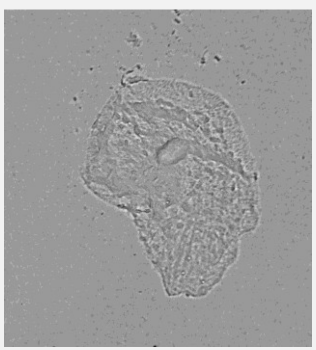
SPC Image*



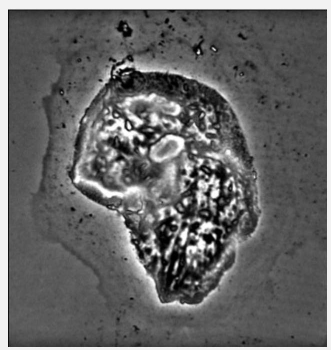
DF Image*

Phase objects

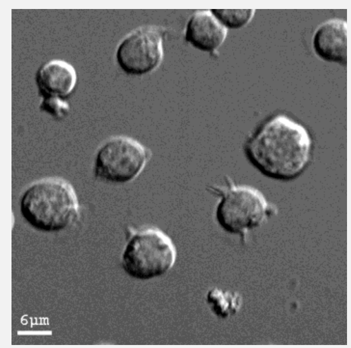
BF Image*



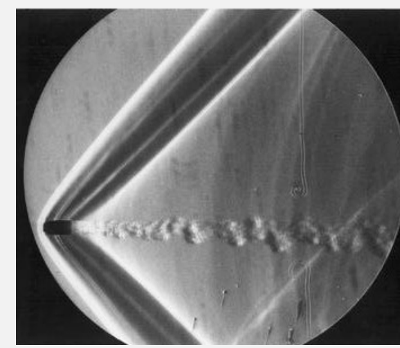
PC Image *



DIC-Image



Schlieren Image

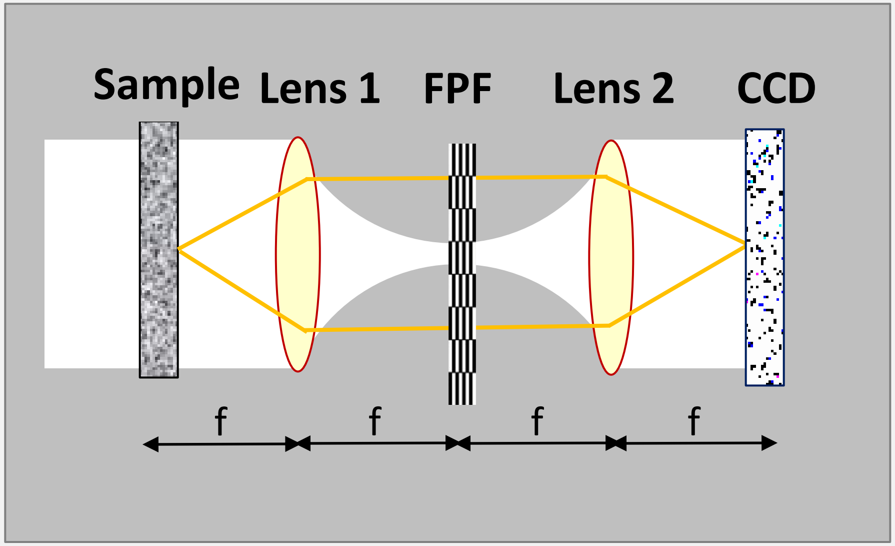


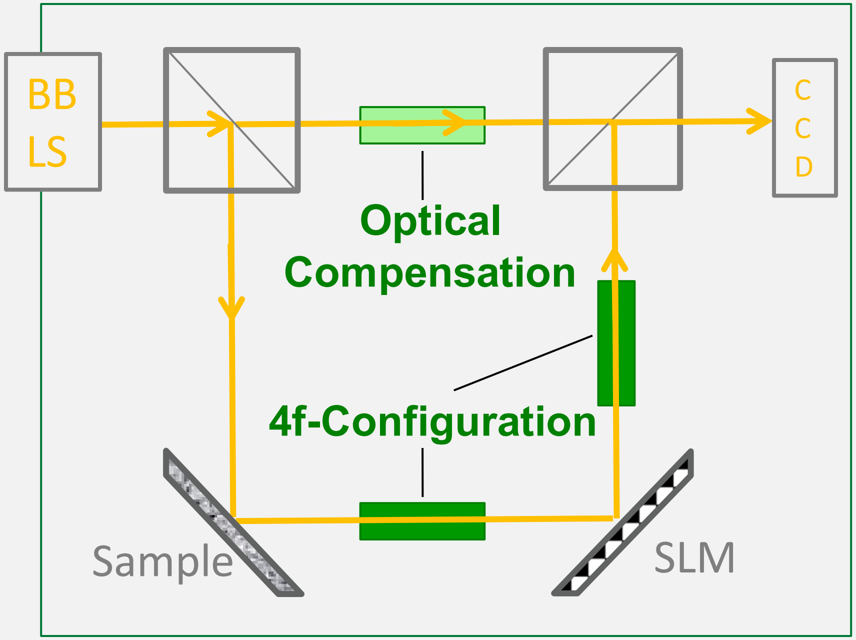
* Images courtesy by Monika Ritsch-Marte

- S. Fuerhapter, M. Ritsch-Marte et al. „ **Spiral phase contrast imaging in microscopy**“ , Opt.Exp (2005)

Imaging I Fourier Plane Filtering (FPF)

FPF can be integrated within low coherence interferometry (LCI)



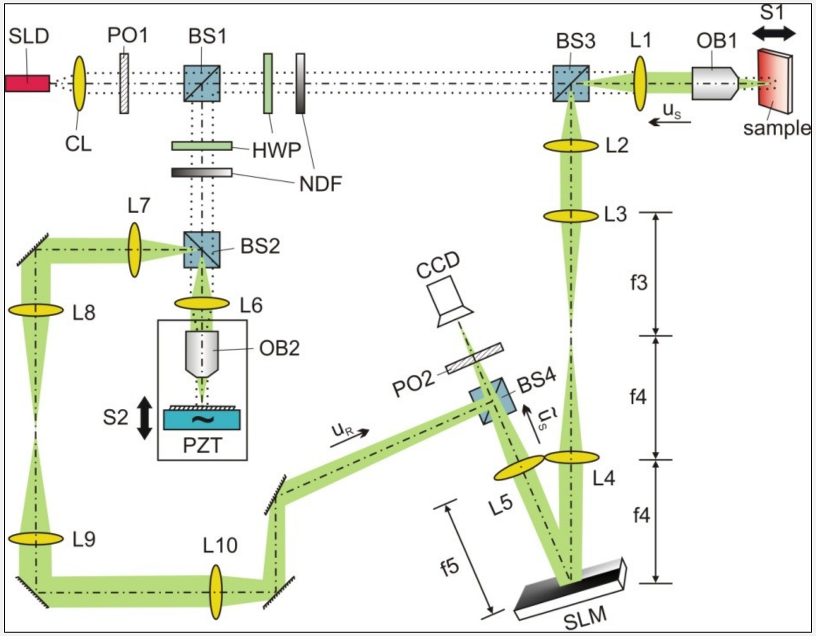


Optical 4f-configuration (transmission)

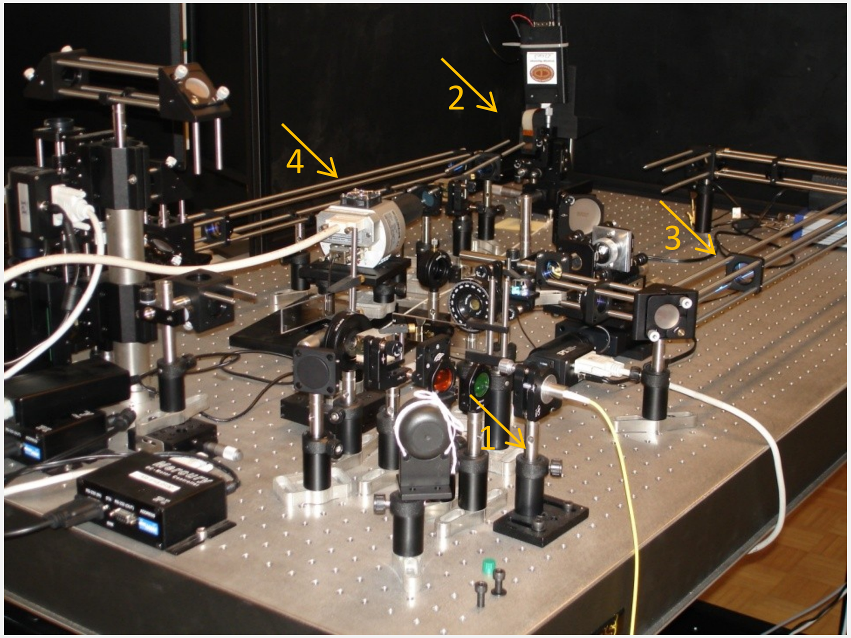
Mach-Zehnder interferometer configuration

Imaging I Fourier Plane Filtering in LCI

FPF can be integrated within low coherence interferometry (LCI)



Scheme of LCI setup with FPF unit in Mach-Zehnder configuration



LCI setup: 1) collimator, 2) FPF unit, 3) reference arm, 4) sample arm

Full- Field Optical Coherence Microscopy (FF-OCM)

Fourier Plane Filtering: Filter types

Fourier plane filters

Phase pattern $\varphi(k_x, k_y)$ can be introduced by height (d) or refractive index (n) changes at optical filter

$$\varphi(k_x, k_y) \sim (2\pi/\lambda)\, n(k_x, k_y)*d(k_x, k_y)$$

glass phase plate

SLM is flexibly addressed by discrete filter functions →

SLM

Imaging | SLM

Spatial Light Modulator (SLM)

- Liquid Crystal Display (LCD)
  - Pixelated LC array
  - Modulate light spatially in each pixel
  - Amplitude, phase, binary SLM versions
  - Transmissive or reflective LC microdisplays
  - Addressable by PC/graphics card

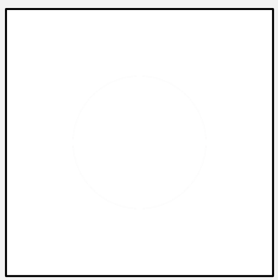
- Pluto Phase-Only SLM (Holoeye)
  - Reflective LCOS micro-display
  - HDTV resolution (1920 x 1080 pixel)
  - 60 Hz image frame rate
  - 8.0 µm pixel pitch
  - 2π phase shift

Imaging I Fourier Plane Filtering (FPF)

Amplitude – Filter function: $H_A(k)$

FPF
Mask
Magnitude

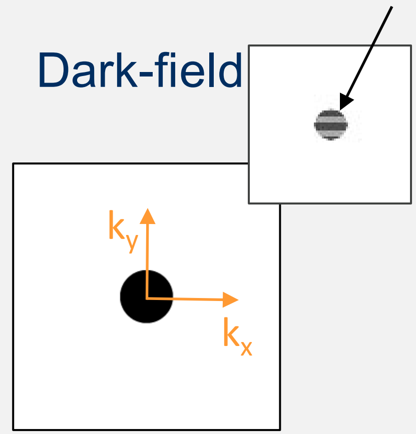
Bright-field



(a)

$$H_A(\mathbf{k}) = 1$$

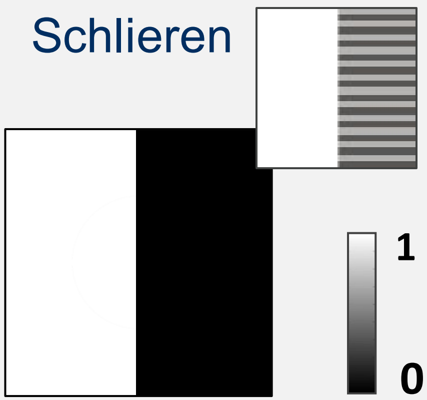
Dark-field



(b)

$$H_A(\mathbf{k}) = \begin{cases} 0 & \text{for } \|\mathbf{k}\| \le k_a \\ 1 & \text{for } \|\mathbf{k}\| > k_a \end{cases}$$

Schlieren



(c)

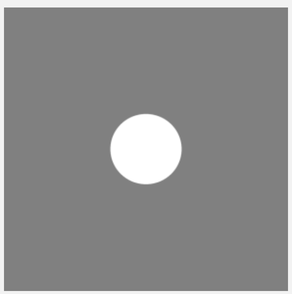
$$H_A(\mathbf{k}) = \frac{1}{2}\left(1 - \operatorname{sgn}(\mathbf{k} \cdot \mathbf{e}_n)\right)$$

Imaging I Fourier Plane Filtering (FPF)

Phase – Filter function: $H_\phi(\mathbf{k})$

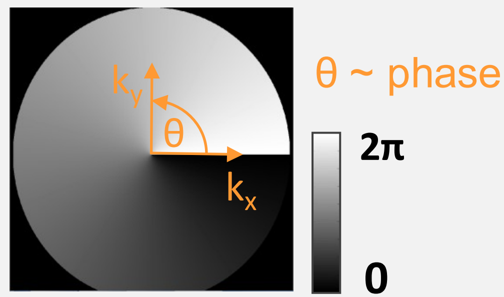
Phase-Contrast

FPF
Mask
Argument

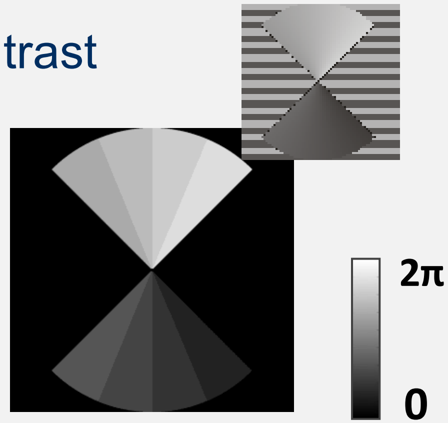


(a)

Spiral Phase Contrast



(b)



(c)

$$H_\Phi(\mathbf{k}) = \begin{cases} \exp(\pm i\pi/2) & \text{for } \|\mathbf{k}\| \le k_a \\ 1 & \text{for } \|\mathbf{k}\| > k_a \end{cases}$$

$$H_\Phi(\mathbf{k}) = -i\frac{\mathbf{k}}{\|\mathbf{k}\|} = \exp[-i\theta(\mathbf{k})]$$

(b) $H_A(\mathbf{k}) = 1 \quad \forall \mathbf{k}$

(c) $H_A(\mathbf{k}) = 1 \quad \forall \mathbf{k} \in \text{cone}$

Signal processing | Monogenic Signal

2D Spatial Domain:

Riesz Transform $\mathcal{R}$ in 2D spatial domain:

Riesz transform kernel $\mathcal{R}$ and its components $\mathcal{R}_1$, $\mathcal{R}_2$

and in polar coordinate (r,θ)

$$\mathcal{R}\{f(\mathbf{x})\} = f_R(\mathbf{x}) = [f_{R1}(\mathbf{x}), f_{R2}(\mathbf{x})] = \frac{\mathbf{x}}{2\pi\|\mathbf{x}\|^3} \otimes f(\mathbf{x})$$

$$= \begin{cases} f_{R1}(\mathbf{x}) = \dfrac{x_1}{2\pi\|\mathbf{x}\|^3} \otimes f(\mathbf{x}) = \dfrac{\cos\theta}{2\pi r^2} \otimes f(\mathbf{x}) \\ f_{R2}(\mathbf{x}) = \dfrac{(i)\,x_2}{2\pi\|\mathbf{x}\|^3} \otimes f(\mathbf{x}) = \dfrac{(i)\sin\theta}{2\pi r^2} \otimes f(\mathbf{x}) \end{cases}$$

$$= (f_{R1}(\mathbf{x}) + i\, f_{R2}(\mathbf{x})) = \frac{\exp(i\theta)}{2\pi r^2} \otimes f(\mathbf{x}), \qquad r > 0$$

with $\mathbf{x} = (x_1, x_2)$, $\exp[i\theta] = \dfrac{\mathbf{x}}{\|\mathbf{x}\|}$, $r^2 = \|\mathbf{x}\|^2$, (i) : complex units

Monogenic Signal

2D Fourier Domain:

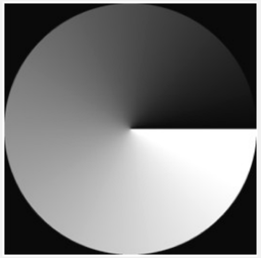
<u>Riesz Transform $\mathcal{R}$^ in 2D Fourier domain:</u>

Riesz transform kernel $\mathcal{R}$^ and components $\mathcal{R}$^$_1$, $\mathcal{R}$^$_2$

$$\hat{\mathcal{R}}\left\{\hat{f}(\mathbf{k})\right\} = \hat{f}_{\hat{R}}(\mathbf{k}) = [\hat{f}_{\hat{R}1}(\mathbf{k}), \hat{f}_{\hat{R}2}(\mathbf{k})] = i\,\frac{\mathbf{k}}{\|\mathbf{k}\|}\cdot\hat{f}(\mathbf{k})$$

and in polar coordinate (r^,θ^)

$$= \begin{cases} \hat{f}_{\hat{R}1}(\mathbf{u}) \\ \hat{f}_{\hat{R}2}(\mathbf{u}) \end{cases} = \begin{cases} (i)\,\dfrac{k_1}{\|\mathbf{k}\|}\cdot\hat{f}(\mathbf{k}) \\ \dfrac{k_2}{\|\mathbf{k}\|}\cdot\hat{f}(\mathbf{k}) \end{cases} = \begin{cases} (-i)\cos\hat{\theta}\cdot\hat{f}(\mathbf{k}) \\ \sin\hat{\theta}\cdot\hat{f}(\mathbf{k}) \end{cases} = \exp(i\,\hat{\theta})\cdot\hat{f}(\mathbf{k})$$



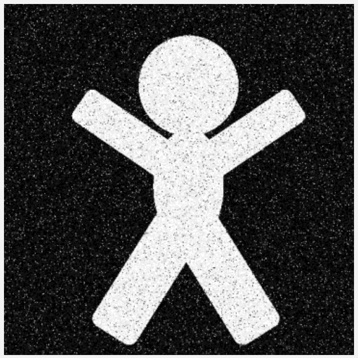
Θ^(k_1,k_2)

$$\text{with}\quad \mathbf{k} = (k_1, k_2),\quad \hat{\theta} = \operatorname{atan}(k_1, k_2),\quad \hat{r} = \|\mathbf{k}\|\ ,\quad (i):\text{complex unit}$$

Spiral phase quadrature filter (Larkin, 2001) ↔ Riesz transform kernel (..., Felsberg, 2001) in Fourier Domain

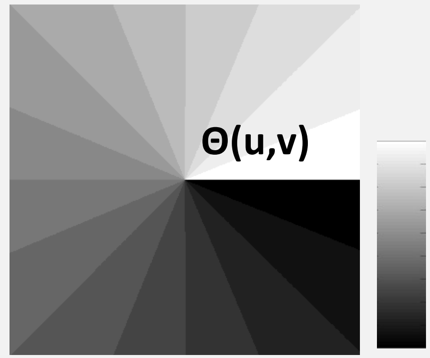
Application: Image Processing

Illustration: Edge enhancement in image processing by applying Riesz tr. kernel



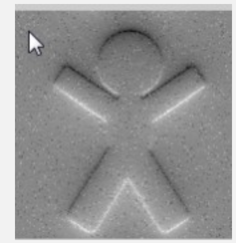
f(x,y)

$\mathcal{F}$

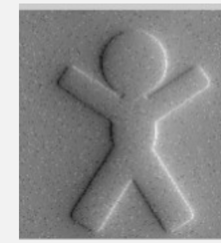


arg[H_R(u,v)]

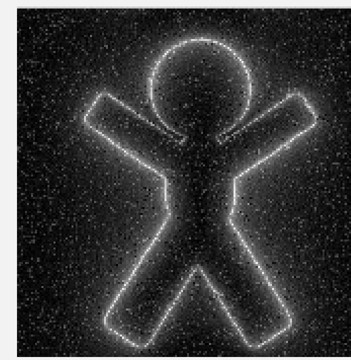
$\mathcal{F}^{-1}$



$f_{R1}(\mathbf{x})$



$f_{R2}(\mathbf{x})$



Magnitude M_R(x,y)

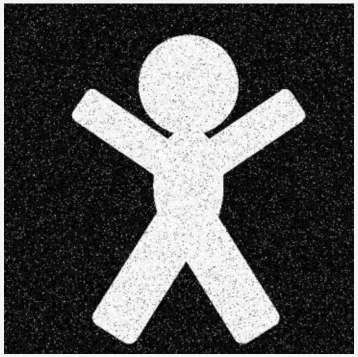
Magnitude (M) of Gradient vs. of Riesz transform components R_1 and R_2 (Energy)

$$M_G : abs(\frac{\partial}{\partial x}\{.\}+i\frac{\partial}{\partial y}\{.\}) \leftrightarrow M_R : abs(R_1\{.\}+iR_2\{.\})$$

- M. Felsberg, G. Sommer, "**Monogenic signal**", 2001
- M. Unser et. al., "**Multiresolution Monogenic signal Analysis Using Riesz Laplace-WT**", (2009)

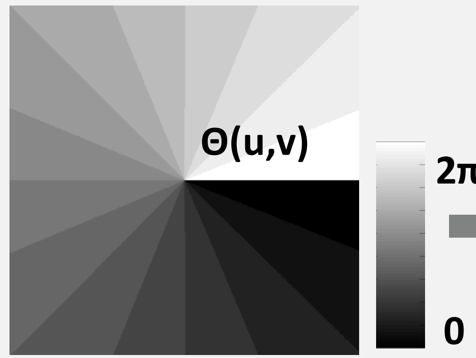
Application: Image Processing

Illustration: Orientation estimation in image processing by applying Riesz tr. kernel



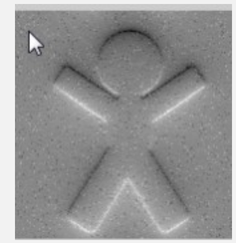
f(x,y)

$\mathcal{F}$

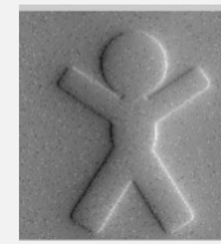


arg[$H_R(k_1,k_2)$]

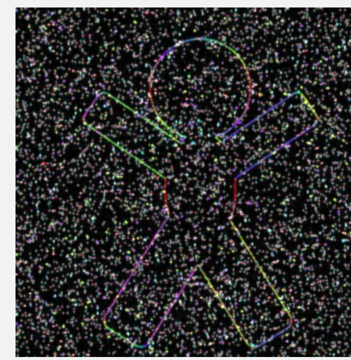
$\mathcal{F}^{-1}$



$f_{R1}(\mathbf{x})$



$f_{R2}(\mathbf{x})$



Orientation $\beta_R(x,y)$

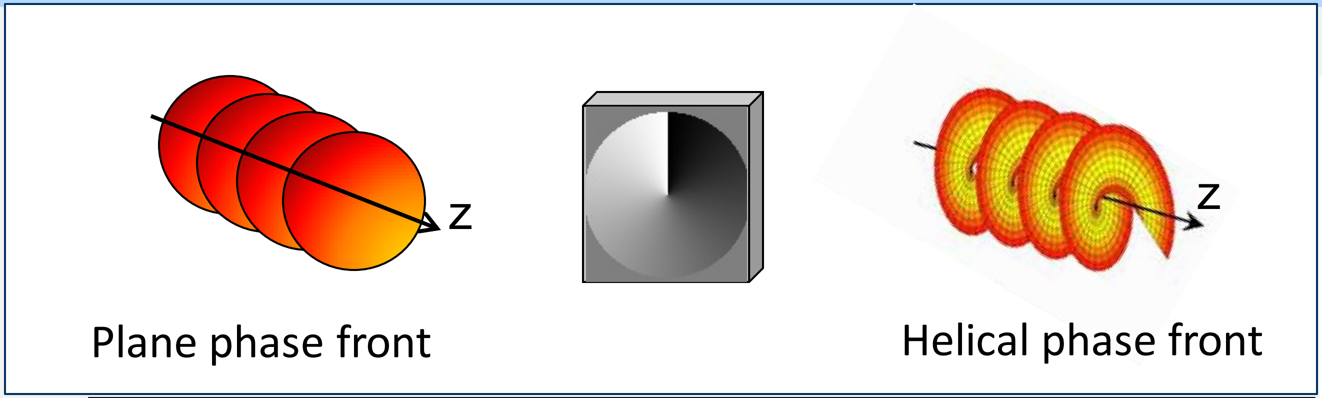
Argument of Gradient vs. **Argument** of Riesz transform (Orientation **β**)

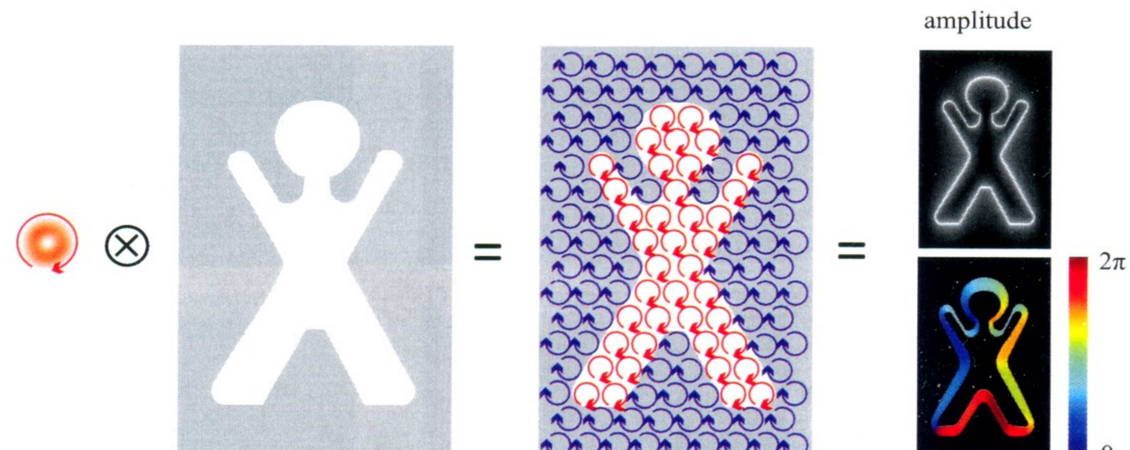
$$\beta_G : \arg(\frac{\partial}{\partial x}\{.\}+i\frac{\partial}{\partial y}\{.\}) \leftrightarrow \beta_R : \arg(R_1\{.\}+iR_2\{.\})$$

- M. Felsberg, G. Sommer, "**Monogenic signal**", 2001
- M. Unser et. al., "**Multiresolution Monogenic signal Analysis Using Riesz Laplace-WT**", (2009)

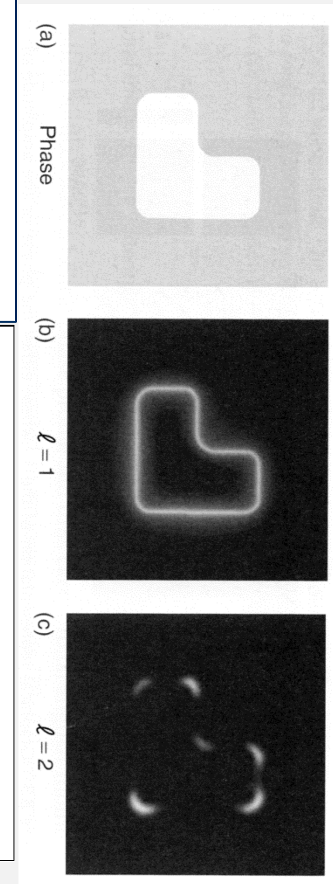
Spiral Phase Filter in Microscopy Imaging

Comparison: Edge enhancement in imaging by spiral phase filtering





$u(x,y) \sim 1/r^2 \exp[i\, l\, \phi(x,y)]$



- Images from: C. Maurer, M. Ritsch-Marte et al., Laser and Photonics review (2010)

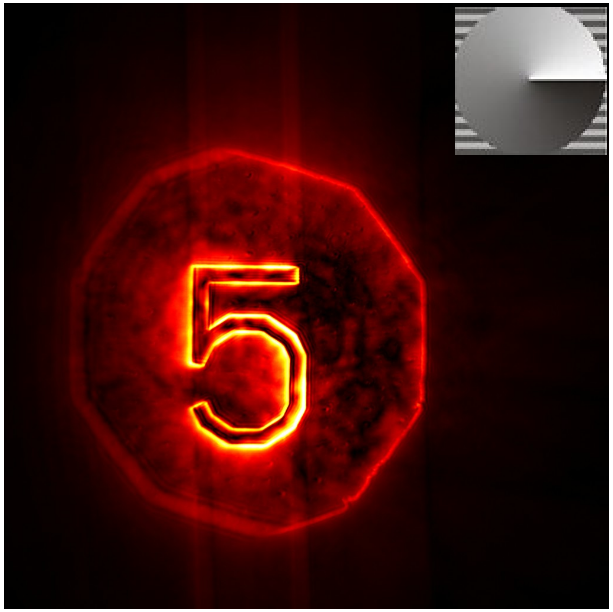
Application: Imaging

<u>Isotropic</u> contrast enhancement by spiral phase filtering (SPF)

Spiral phase (quadrature) filter (SPF) in physics &
(2D) Riesz transform kernel (RT) in mathematics

Bright field

SPF



Phase only filter function

$$H_A(\mathbf{k}) = 1;\ \ H_\Phi(\mathbf{k}) = \exp[i\ l\theta(\mathbf{k})],\ \ l = 1,\ \ \theta(\mathbf{k}) = \angle\mathbf{k}$$

Microscopic imaging

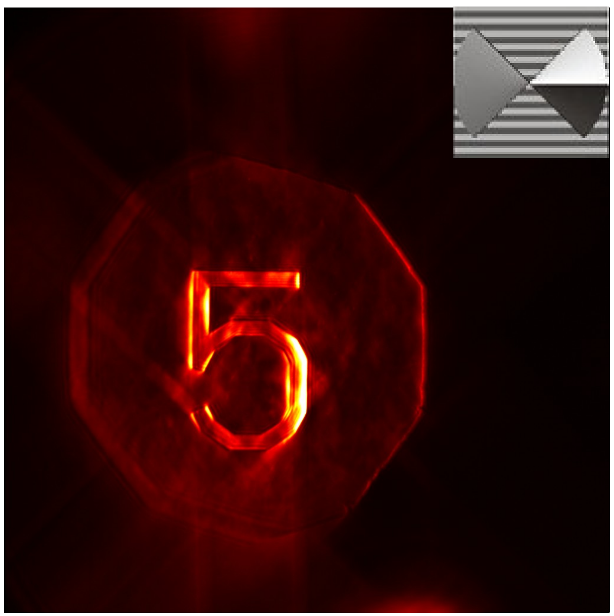
On axis configuration

Application: Imaging

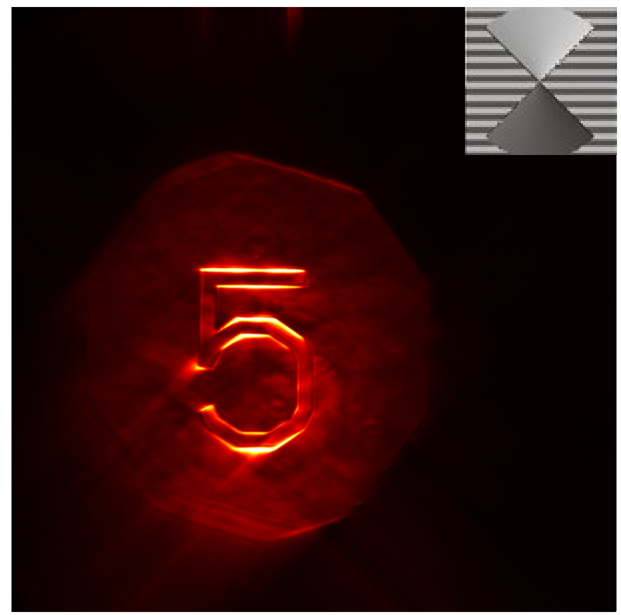
Anisotropic contrast enhancement by modified SPF

Cone-like filters (“curvelets”) & SPF/ Riesz transform kernel (phase)

Directional cones: vertical structures



Directional cones: horizontal structures



$$H_A(\mathbf{k}) = 1,\ \mathbf{k} \in cone, \quad H_\Phi(\mathbf{k}) = \exp[i\ \theta], \quad l = 1$$

Microscopic imaging

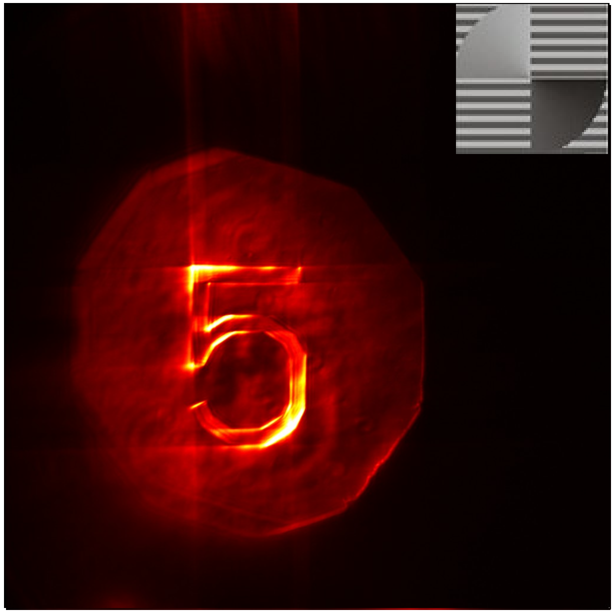
On axis configuration

Application: Imaging

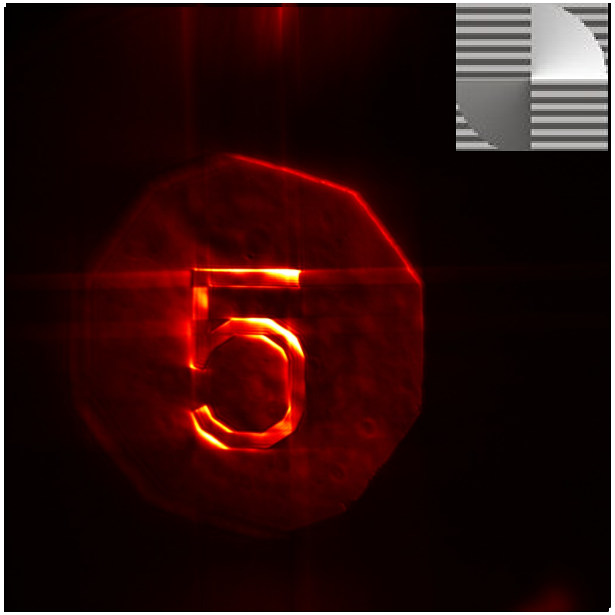
Anisotropic contrast enhancement by modified SPF

Cone-like filters (“curvelets”) & Riesz transform kernel (phase)

Directional cones:
diagonal structures



Directional cones:
diagonal structures



$$H_A(\mathbf{k}) = 1,\ \ \mathbf{k} \in cone, \quad H_\Phi(\mathbf{k}) = \exp[i\ \theta], \quad l = 1$$

Microscopic imaging

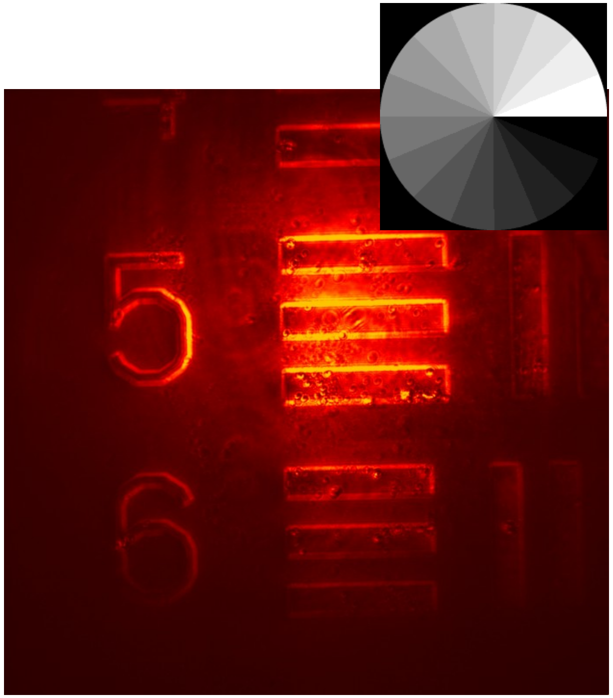
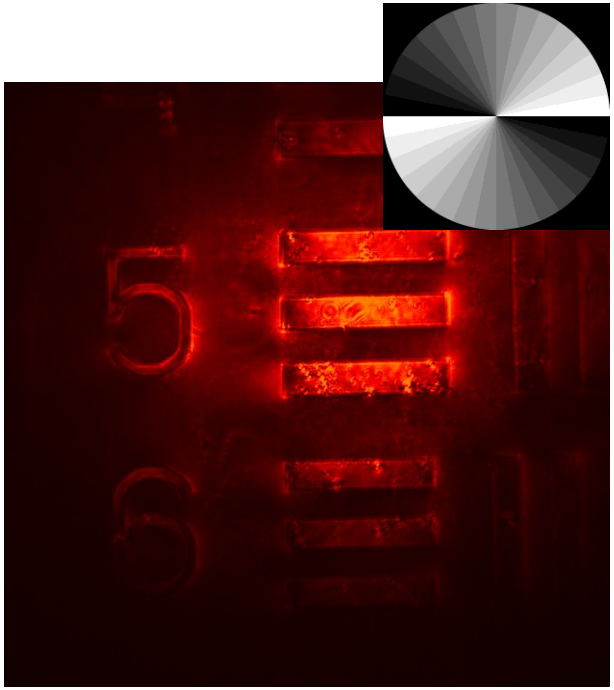
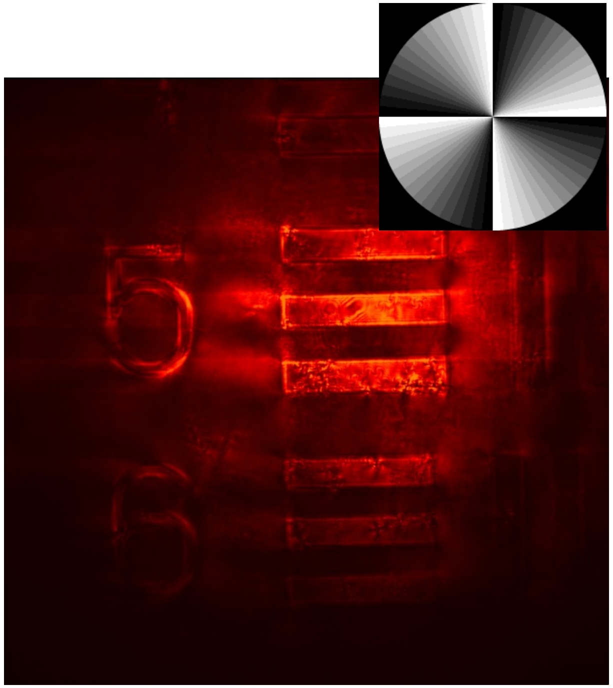
On axis configuration

- Modifications of SPF: Higher order SPF

  Fractional SPF

Application: Imaging

Higher order spiral phase filter

l=1 l=2 l=4

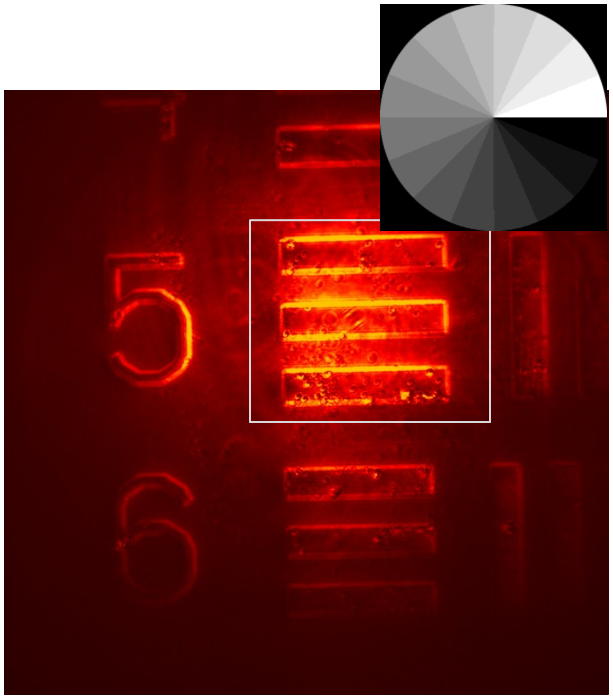
$$H_\Phi = \exp[i\, l\, \theta], \quad l \in N^+$$

On axis configuration

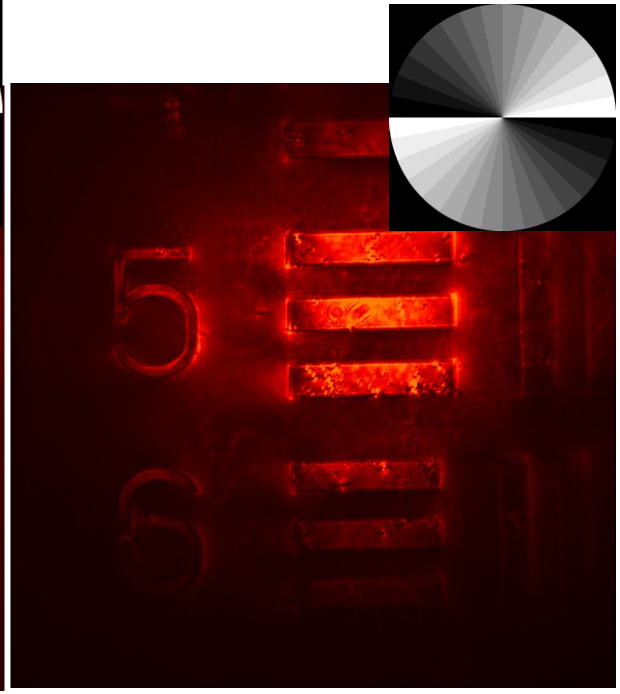
Microscopic imaging

Application: Imaging

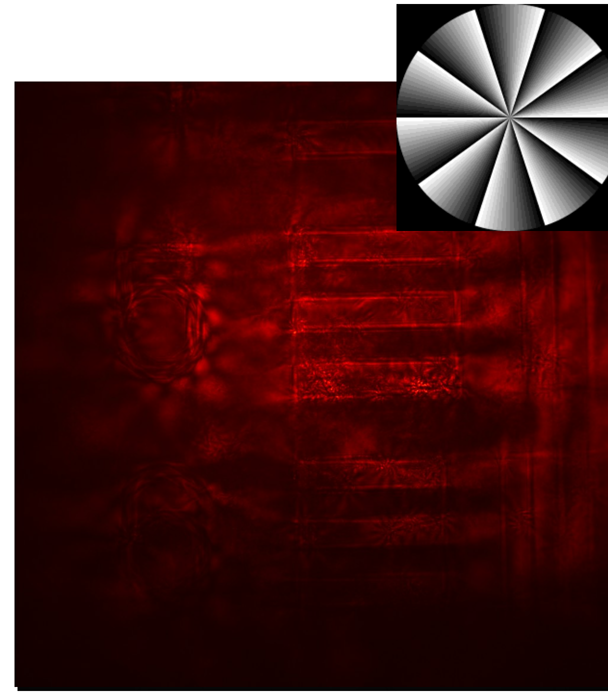
Higher order spiral phase filter/ Modified RT?



l=1



l=2



l=10

$$H_{\Phi} = \exp[i\,l\,\theta], \quad l \in N^{+}$$

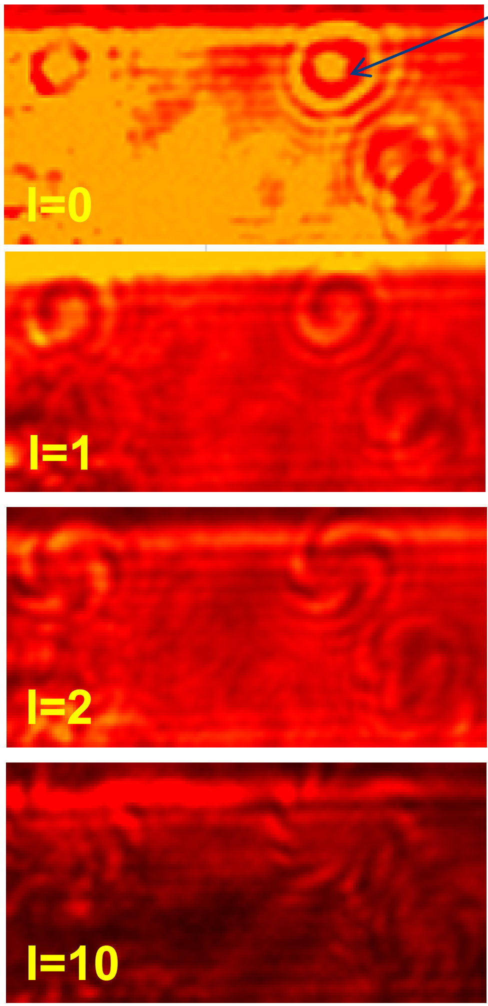
On axis configuration

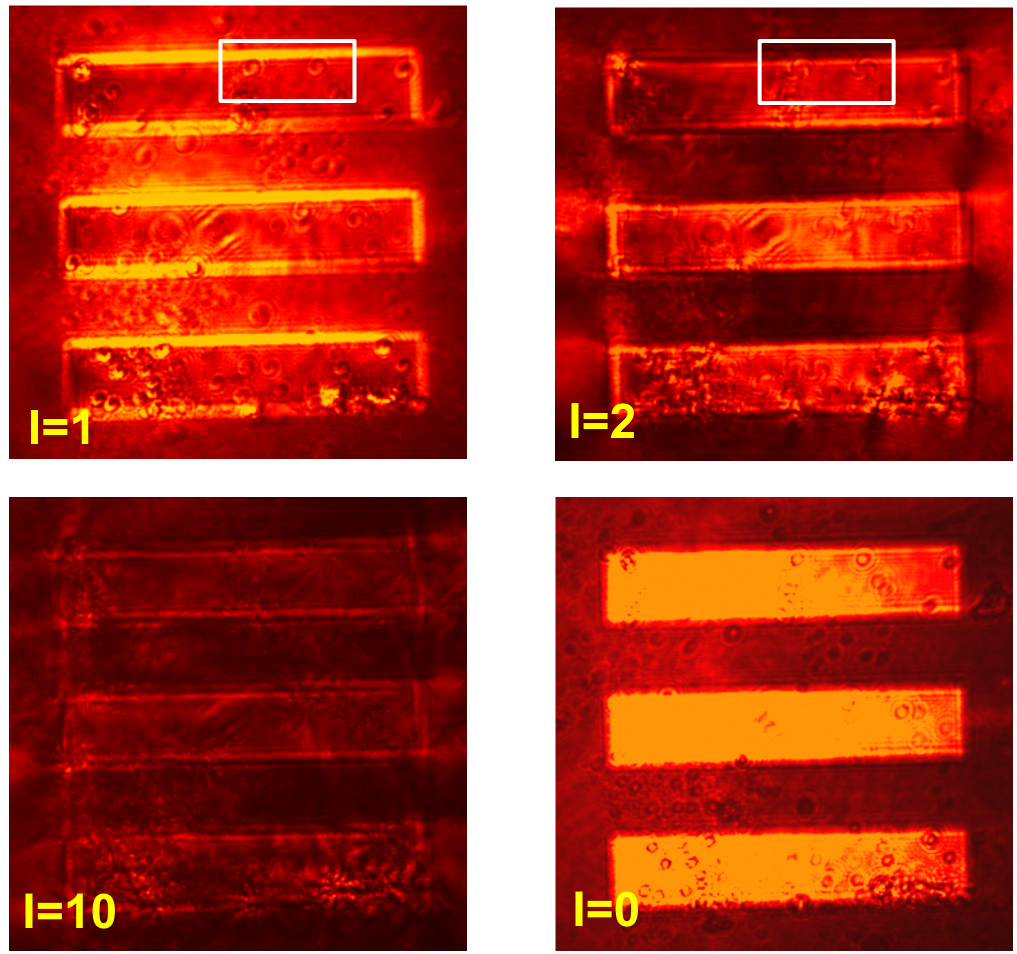
Microscopic imaging

Application: Imaging

Generation of spiral fringes of different topological charges

Dust particle (rings: interference effects)





Application: Imaging

Fractional order spiral phase filter / Fractional RT?/ ...?

l=0.5

l=1

$$H_{\Phi} = \exp[i\, l\, \theta], \quad l \in Q$$

Microscopic imaging

On axis configuration

- Modification: off axis FPF configuration

Application: Ongoing Work: Off axis Imaging

Fourier plane filter in off axis configuration

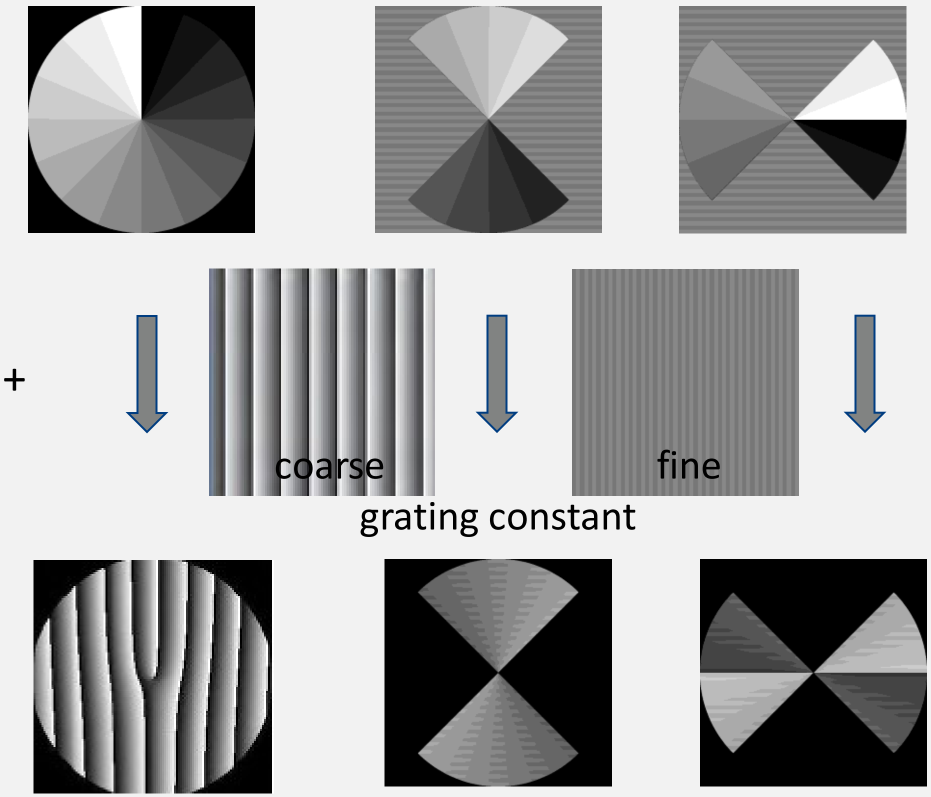
On axis configuration

Direct reflection

additional grating +

Off axis configuration

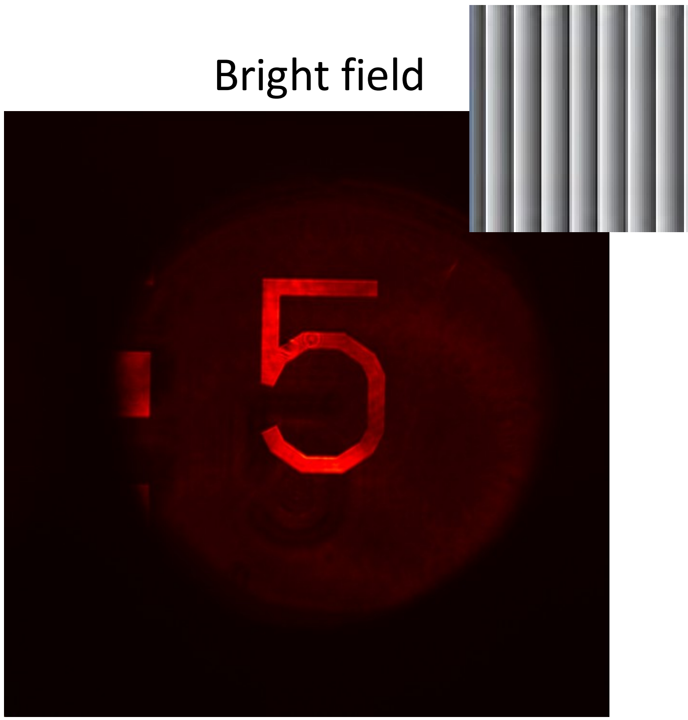
First diffraction order



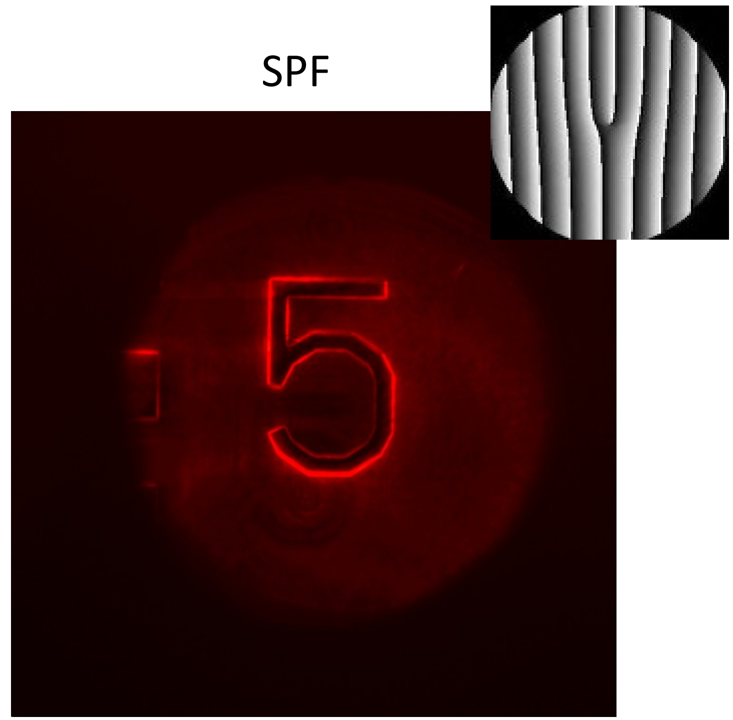
Application: Ongoing Work: Off axis Imaging

Isotropic contrast enhancement by Spiral phase filtering (SPF) / RT

Bright field



SPF



$$H_{\Phi} = \exp[i\ \theta]$$

Off axis configuration

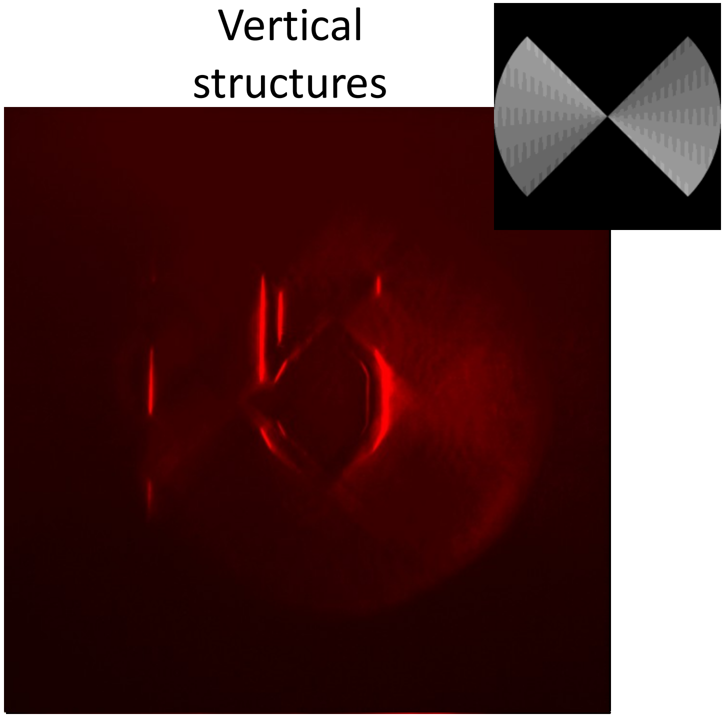
Microscopic imaging

Application: Ongoing Work: Off axis Imaging

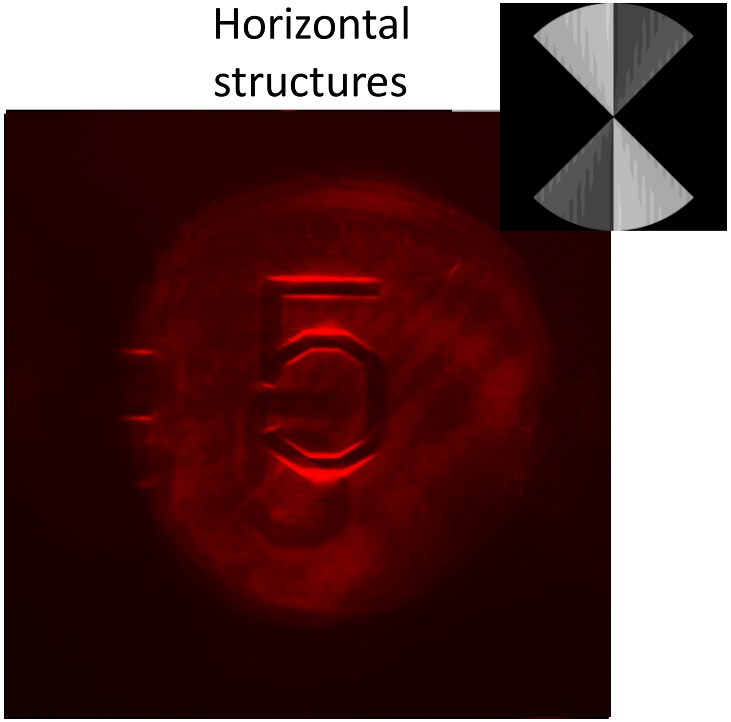
Anisotropic contrast enhancement by modified SPF

Cone-like filters & Riesz Transform

Vertical structures



Horizontal structures

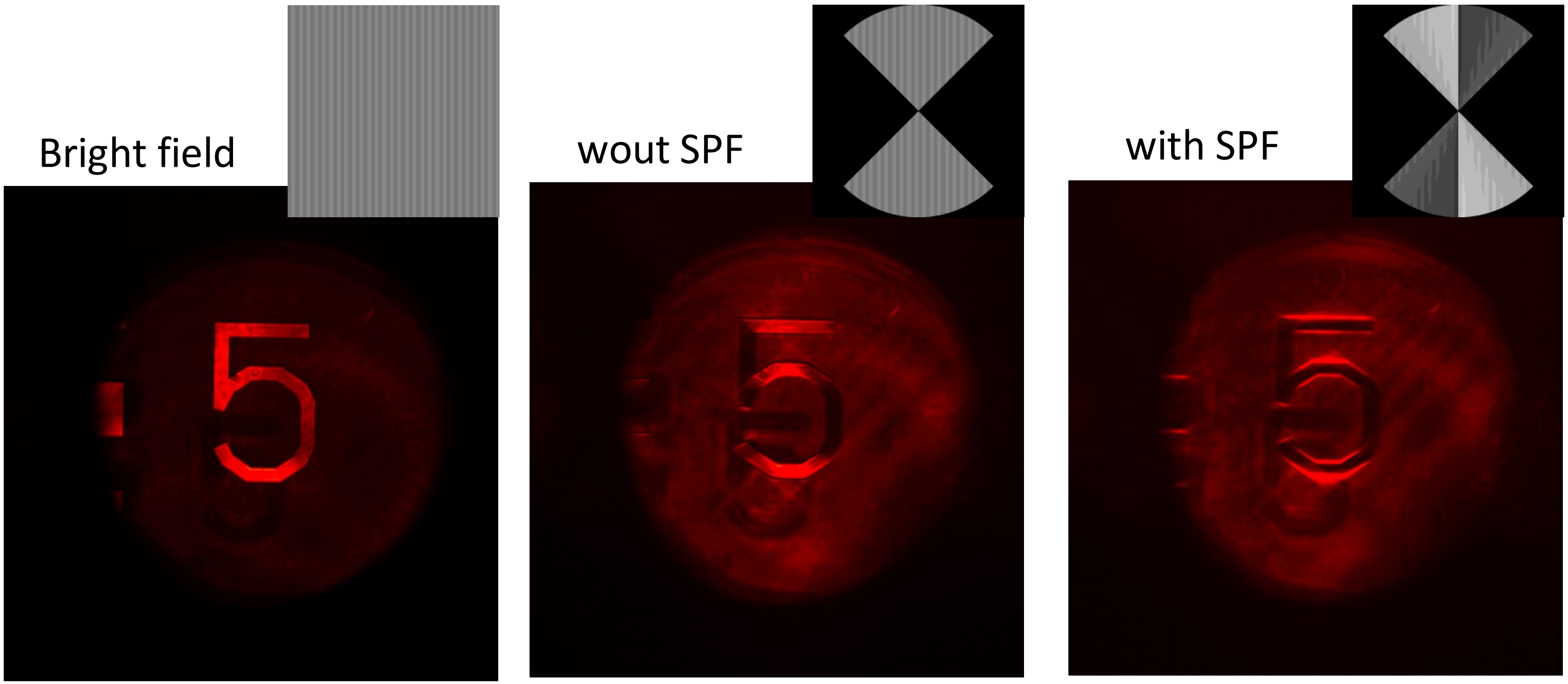


<u>Off axis </u>configuration

Microscopic imaging

Application: Ongoing Work: Off axis Imaging

Comparison: Cone-like filters : wout vs.with spiral phase component



Off axis configuration

Microscopic imaging

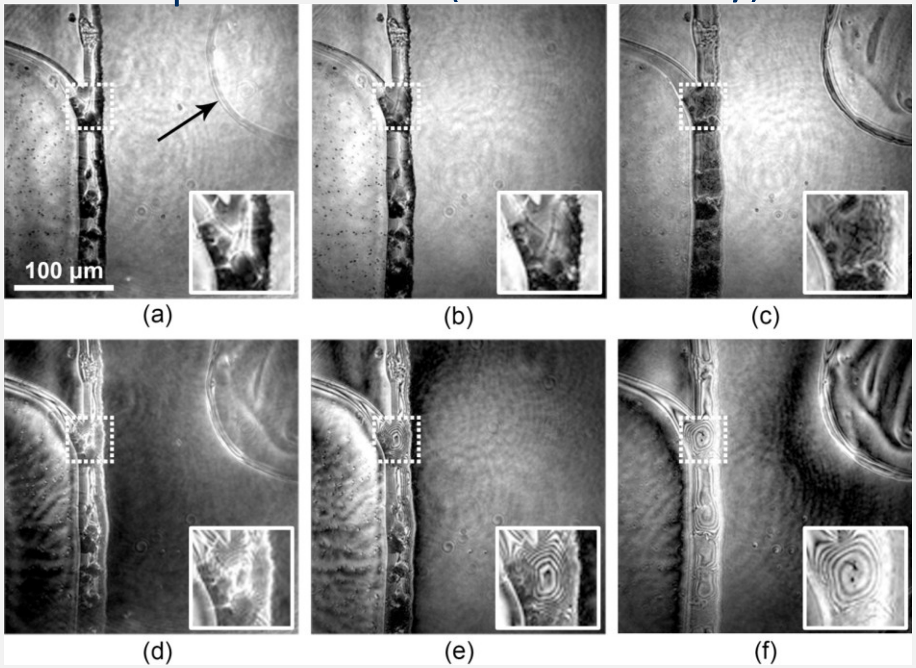
- Modification: interferometric configuration

Imaging I Contrast Modification in FF-OCM by FPF

Contrast modification by FF-OCM imaging <u>within</u> transparent sample

Microscopy setup

Optical Coherent Microscopy (Interferometry)



Conventional BF Contrast

Modified SPF Contrast

both features

'line' feature

Surface

'dot' feature

Backside

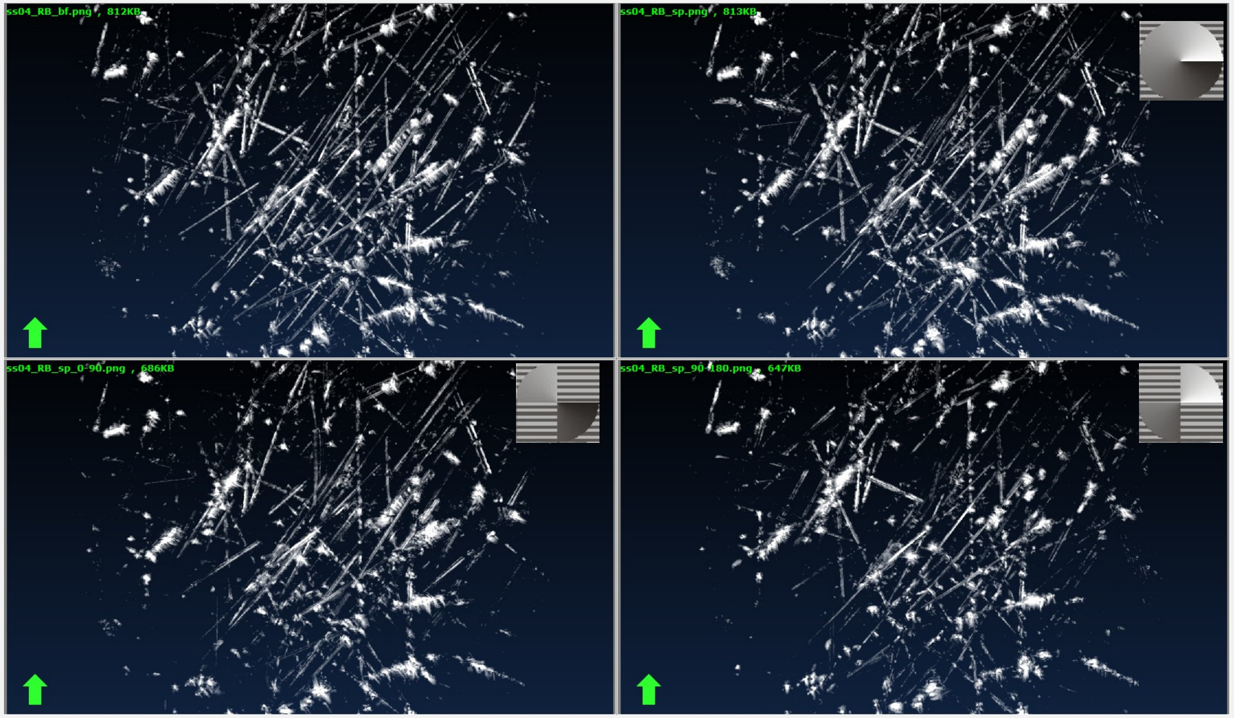
Sample: polymeric features on glass slide

<u>Interferometric imaging</u>

➢Schausberger, S.E., et al., Opt. Letters 35, (2010).

Imaging I Contrast Modification in FF-OCM by FPF

Contrast modification by FF-OCM imaging <u>within</u> scattering sample



Sample: glass-fiber reinforced polymer

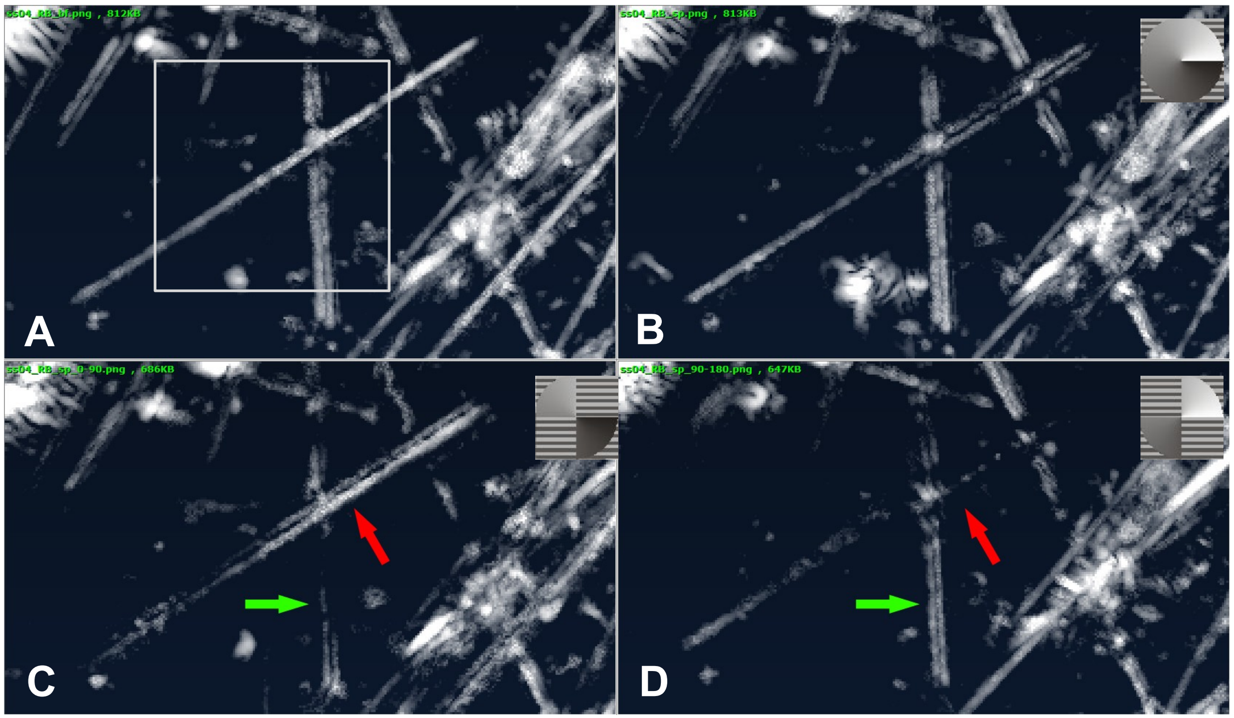
<u>Interferometric imaging</u>

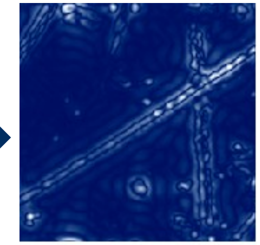
➢Schausberger, S.E., et al., Proc. SPIE, (2011).

Imaging I Contrast Modification in FF-OCM by FPF

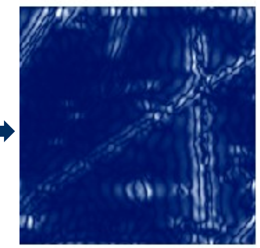
shearlet coefficient images

Comparison: Optical vs. Mathematical filtering for (low coherence) interferometric data

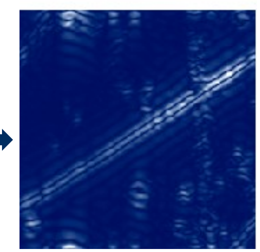




all coeff. 2^{nd} sc.



hor coeff. 2^{nd} sc.



ver coeff. 2^{nd} sc

<u>Optical</u> FP filtering

within slightly scattering fiber material (interferometric setup)

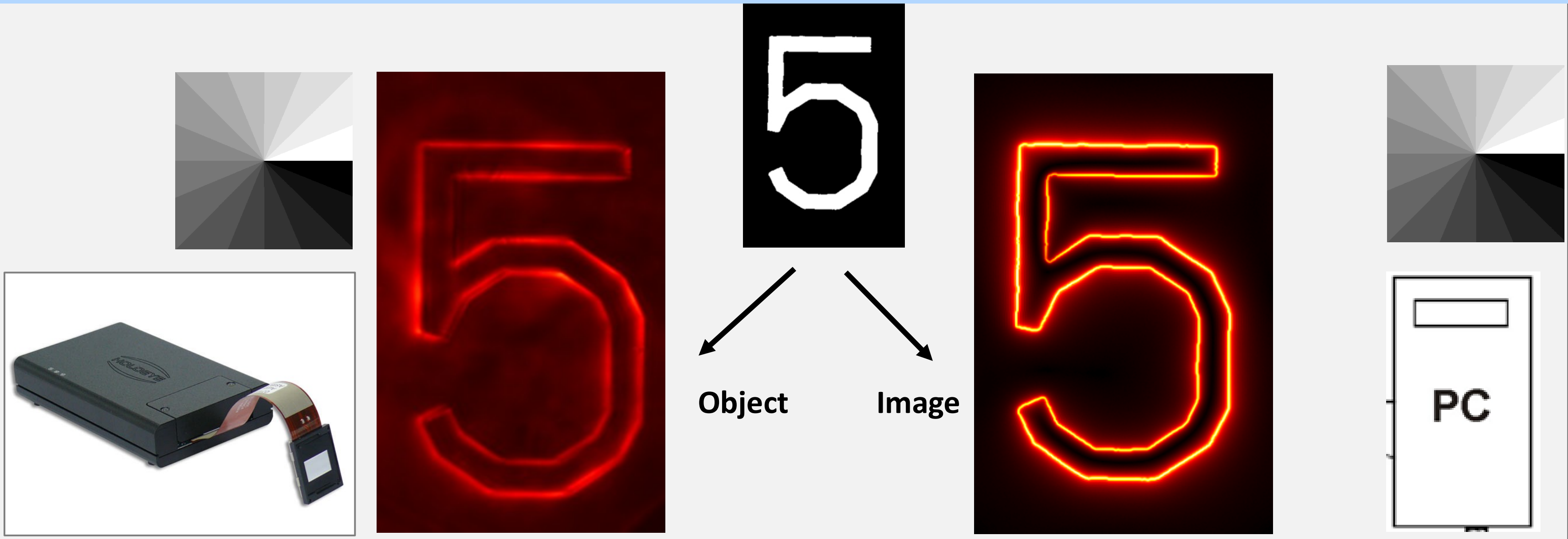
<u>Mathematical</u> shearlet filtering:

using FFST Toolbox, S. Häuser, Uni Kaiserslautern

- Summary & Outlook

Summary: FPF in Imaging & Image Processing

Comparison: Optical vs. Mathematical filtering



Optical Imaging

Focal plane:

Spiral phase filter (SPF)

Mathematical Image Processing

Fourier domain:

Riesz transform (RT)

SPF & RT: Optical Fourier/ wavelet filters in imaging vs. mathematical Fourier/ wavelet filters in image processing lead to similar results (e.g. edge enhancement), ...but keep in mind ...

Summary & Outlook

Restrictions & Potential for SLM technique in FPF and contrast modification

- Pixelation and discretization of SLM array (1920x1080, 10 µm, 8bit)
- Phase-only array
- Frame rate (60Hz)

→ Technology improvement

- Contrast modification within scattering material

→ Computational techniques: Focusing through scattering media

- Single filter component

→Multiplexed filtering

→Multiscale analysis

- Intensity based measurements, phase ?, synthesis?

→Phase retrieval

References

- Felsberg M, Sommer G. *The monogenic signal*. IEEE Trans. Sign. Proc. 2001; 49(12), 3136-3144.

.

- Larkin KG, Bone DJ, Oldfield MA. *Natural demodulation of two-dimensional fringe patterns. I. General background of the spiral phase quadrature transform*. J. Opt. Soc. Am. A 2001; 18(8), 1862-1870.
- Maurer C, Jesacher A, Bernet S, Ritsch-Marte M. *What spatial light modulators can do for optical microscopy*. Laser & Photonics Reviews 2011; 5(1), 81-101.
- Yao AM, Padgett MJ. *Orbital angular momentum: origins, behavior and applications*. Advances in Optics and Photonics 3, 161–204 (2011), doi:10.1364/AOP.3.000161.
- Berry MV, *Optical vortices evolving from helicoidal integer and fractional phase steps*. J. Opt. A: Pure Appl. Opt. 2004; 6, 259–268.
- Schausberger SE, Heise B, Maurer, M Bernet, S, Ritsch-Marte M, Stifter D. *Flexible contrast for low-coherence interference microscopy by Fourier-plane filtering with a spatial light modulator*. Opt. Letters 35, 4154-4156 (2010).
- Heise B, Schausberger SE, Stifter D. *Coherence Probe Microscopy Contrast Modification and Image Enhancement*. Imaging & Microscopy 2012; 2, 29-32.
- Heise B, Schausberger SE, Maurer C, Ritsch-Marte M, Bernet S, Stifter D. *Enhancing of structures in coherence probe microscopy imaging*. Proc. SPIE 8335, 83350G-1-83350G-7 (2012).

Thanks to:

- ***ZONA at JKU Linz***
- ***FLLL at JKU Linz***
- ***Biomed. Physics at Med-Uni Innsbruck***
- ***Erasmus program (BA Freiberg)***

ACKNOWLEDGMENTS

Christian Doppler Gesellschaft

CDL MS-MACH

Questions?